Inter-temporal income polarization

Francesca Subioli[1] Philippe Van Kerm[23]

January 2023

Preliminary draft, please do not circulate

Abstract

This paper re-defines Esteban and Ray's (*Econometrica*, 1994) income polarization index to incorporate a role for time. Income polarization captures the extent to which an income distribution concentrates around two or more income levels. Polarization measurement is rationalized as measuring potential conflict in a society where people feel alienated from one another when distant in income but identified with others of similar income levels. We introduce time in this model because the two key ingredients of polarization—alienation and identification—have fewer implications for potential conflict if individual incomes vary over time and feelings of alienation or identification have limited time to form and consolidate. Accordingly, we propose an inter-temporal income polarization measure using panel data in which memory parameters allow past income differences to determine the degree of alienation and identification in a society's income distribution. This leads to measures of income polarization that are sensitive to the history of interpersonal income proximity and distances in past income trajectories. We illustrate the empirical relevance of this longitudinal perspective with an application to Italian data.

JEL Codes: D31, D63
Keywords: Income polarization, income mobility, alienation, earnings dynamics, income inequality, Italy

We are grateful to the Giacomo Brodolini Foundation for allowing us to use the AD-SILC dataset for this project, and to Michele Raitano (Sapienza University of Rome) for the precious support in using it correctly. All mistakes are our responsibility. We also thank the Department of Social Sciences of the University of Luxembourg for hosting Francesca Subioli and funding the visit that gave rise to this work. The paper benefited from comments and suggestions on previous versions from participants at the 4th International Conference on European Studies, the 7th Annual Symposium of the Society for Economic Measurement, the 37th Annual Conference of the Italian Association of Labor Economics, and members of the INEQ group at Sapienza University of Rome.

[1]Department of Economics and Law, Sapienza University of Rome, francesca.subioli@uniroma1.it
[2]Luxembourg Institute of Socio-Economic Research (LISER), philippe.vankerm@liser.lu
[3]Department of Social Sciences, University of Luxembourg, philippe.vankerm@uni.lu

1 Introduction

Income polarization is the tendency of an income distribution to cluster around distinct income levels: individuals belonging to the same cluster share similar levels of income which are (perceived to be) clearly distinct from incomes of individuals located around other poles. The polarization of an income distribution combines elements of equality and inequality (Esteban and Ray, 1999): intra-group homogeneity and inter-group heterogeneity are the two key features of a polarized society.

The economic literature has developed two main methods of measuring polarization based on alternative conceptualisations of polarization itself. In response to a concern about a shrinking middle class in the U.S. society at the beginning of the 1990s, one method views polarization as the hollowing out of the middle of the income distribution towards the tails. This strand of literature started with the work of Wolfson and Foster (Wolfson, 1994, 1997; Foster and Wolfson, 2010) and is referred to as *bi-polarization* – typically across both sides of the median. Esteban and Ray developed an alternative conceptualization of income polarization as the tendency of the income distribution to concentrate around two or more poles, irrespective of where they are located (Esteban and Ray, 1994; Duclos et al., 2004; Esteban et al., 2007). They rationalize income polarization as a measure of potential conflict in a population, relating it to feelings of alienation that people feel from one another when distant in the population, and to the strength of identification that people feel toward people close by. Within this identification-alienation framework, the society is seen as made of groups which are internally homogeneous and externally heterogeneous according to a relevant characteristic: in the case of income polarization, people share similar levels of income with those belonging to the same group – with whom they feel identified – while they are far from the members of other groups – over whom they feel alienated.

Esteban and Ray (1994) refer to the Marxian theory of classes to highlight the relevance of polarization beyond inequality:

> «We begin with the obvious question: why are we interested in polarization? It is our contention that the phenomenon of polarization is closely linked to the generation of tensions, to the possibilities of articulated rebellion and revolt, and to the existence of social unrest in general.» (Esteban and Ray (1994), p. 820).

The main intuition is that polarization is related to the *effective antagonism* between individuals in a society. Such antagonism is the result of both the alienation that people feel from one another – monotonic in the absolute distance as in the classical conceptualization of inequality – and the strength of *group identity* – function of the relative size of income groups. For alienation to become effective voice, action or protest, the individual must not be alone but rather identify with others (Esteban and Ray, 1994). This makes polarization different from inequality.

Existing operationalization of the notion of polarization and empirical work have discarded any time dimension: income polarization is assessed on a snapshot of the income distribution in a cross-sectional perspective. In our view, this is unsatisfactory: we argue

here that the two key ingredients of polarization—feelings of alienation and identification—are sensitive to the duration of individuals' proximity or distance. Duclos et al. (2004) point out that large-scale social unrest—strikes, demonstrations, widespread violence, revolts—are phenomena that thrive on differences but cannot exist without notions of group identity. In our view, group identity and effective antagonism are not transient phenomena: it takes time to create bonds and break them, and to develop, consolidate, and translate feelings of alienation and identification into collective action. High (income) polarization at a point in time may therefore have much fewer implications for potential conflict if individuals move around the income distribution over time than if the feelings of identification and alienation consolidate through persistent income proximity or distance.

This issue is reminiscent of the literature on poverty dynamics which distinguishes snapshot – one period – poverty from a chronic or persistent poverty status (see, among others, Calvo and Dercon, 2009; Bossert et al., 2012; Hoy et al., 2012). On similar premises, we propose a generalization of the Esteban and Ray (1994) polarization index in a temporal perspective: by introducing memory parameters of past income differences, our inter-temporal polarization index measures the concentration around poles of income *trajectories* rather than point-in-time income values. By defining proximity and distance as the closeness and the remoteness of income paths, this procedure allows the dynamics of income to mediate the identification-alienation mechanism. A parameter of memory allows to calibrate the degree of relevance of the past; setting the parameter to zero, we obtain the standard Esteban and Ray index as a limit case. This property implies that when people have no memory of their past income and of that of others, polarization can be computed on current income. On the contrary, in case people remember how much they earned before, the resulting polarization value is based on differences between income trajectories.

We apply this longitudinal perspective to matched survey-administrative data for Italy using a cohort approach, comparing income vectors of people of the same age. We document a long-term trend of increasing alienation but decreasing identification which leads to a picture of declining polarization over time. We show that incorporating past differences affects proximity and distance differently. In the identification process, it lowers the number of people belonging to the same group, distancing from each other people who would be closer in terms of current income. Conversely, alienation between people is mitigated when computed on income trajectories, suggesting a role for income mobility in reducing long-run distances between people who would instead be far apart. The effect of allowing for memory is not the same across cohorts, as they differ in their income dynamics patterns. We also demonstrate how important it is to take into account zero earners in the measurement of polarization for the categories of the population more exposed to non-employment risk, and show that the level of effective antagonism experienced is linked to some socio-demographic characteristics.

We begin by setting notation and formally describing the original approach of Esteban and Ray (1994) and its later developments in Section 2. The inter-temporal polarization

measure is developed in Section 3: we first derive and characterize a measure of interpersonal income distance between income trajectories in Section 3.1, and then use this model to develop our inter-temporal polarization index in Section 3.2. Section 4 discusses estimation issues. Section 5 provides an application to cohorts of Italian workers. Section 6 concludes.

2 The notion and measurement of polarization

Esteban and Ray (1994) (henceforth ER) model income polarization as the average effective antagonism in a society, where effective antagonism is a function of identification and alienation feelings of the members of different income groups. Initially developed for an income distribution represented by a finite set of discrete income classes and levels (Esteban and Ray, 1994), implementations of ER's model on micro-data now usually rely on continuous extensions thereof (Duclos et al., 2004; Esteban et al., 2007). We describe the discrete case for clarity of exposition but will expand on the continuous representations in the rest of the paper.

Polarization over discrete income classes With n (discrete) income groups with income levels $\{y_i\}_{i=1}^n$, ER express the antagonism T as a function of alienation A that depends on pairwise income distances $\delta(y_i, y_j)$, and identification I, that is a function of the proportion π_i of the population at the income level y_i

$$P(\pi, \mathbf{y}) = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i \pi_j T(I(\pi_i), A(\delta(y_i, y_j))). \tag{1}$$

Total polarization, therefore, depends both on the distance between income groups $\delta(y_i, y_j))$ and on their relative size π_i.

Three axiomatic restrictions lead ER to model identification as the group relative numerosity itself, and alienation as the average absolute income distance, ending up, in an additive utilitarian context, with the following functional form for polarization:

$$P^{\text{ER94}}(\pi, \mathbf{y}) = K \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i^{1+\alpha} \pi_j |y_i - y_j| \tag{2}$$

where K is a positive constant and $\alpha \in [1, 1.6]$ is a parameter measuring the degree of sensitivity to identification of the index. The larger α, the more polarization departs from inequality by giving more weight to the group size component of the index.[1] This can be easily seen by comparing the index in Equation (2) and the Gini index: when $\alpha = 0$ and $K = \frac{1}{2\mu}$ (where μ is average income), Equation (2) measures half the average (mean-normalized) income distance in the population, which is exactly the Gini inequality index.

[1] The boundaries for the parameter α are derived in Esteban and Ray (1994), p. 833.

In summary, as explained in Duclos et al. (2004), the level of polarization depends on both the separate contributions of alienation and identification and on their co-movement. Increased alienation comes from larger income distances, while increased identification is due to population shifts from less crowded to more crowded groups. The final effect depends on the product of the two components.

From discrete groups to continuously measured incomes The original model of ER works with discrete income classes like quintile groups, assuming the income distribution to be pre-arranged in mutually exclusive groups. However, allowing identification only inside income groups causes a discontinuity problem at the boundaries: comparing two people who are very close to a threshold but on opposite sides of it leads to a violation of the assumption that the groups are internally homogeneous and externally heterogeneous. Moreover, it requires to believe that the pre-arranged grouping conforms with the psychological group identification process (Esteban et al., 2007).

To address this issue, ER propose an extension consistent with their axioms but which avoids arbitrary discretisation of incomes and discontinuities. Individuals with income y_i have a window of identification centred on y_i within which they perceive other individuals as neighbours, feeling identified with them. Each individual, therefore, has his or her own set of identification. Possibly, feelings of identification to i may be strongest with individuals j having exactly the same income ($y_i = y_j$) and weaker with individuals whose income y_j is close to the boundaries of the window. This suggests the use of weights decreasing with distance to model the identification process around individual income. This approach shifts the grouping rationale from splitting the distribution into non-overlapping groups to allowing rolling individual identification windows.

To formalize this, let $b > 0$ be an amount of money such that if an income y is within b of an income y' there is some identification between two persons earning y and y'.[2] Then, let $w(d; b)$ be a positive weighting scheme on $[0, b]$ that decreases with the distance $d \equiv |y - y'|$ and reaches zero at $d \equiv |y - y'| = b$. Outside the window bounded by $y \pm b$, the weight is always zero.

Moving from a discrete set of income groups to a rolling, individual-level identification window allows re-expressing the polarization measure over continuously distributed incomes. If F denotes the continuous income distribution function, then the extent of identification at y is given by the continuous sum of the weights over all the other income levels y':

$$I(y; F, b) = \int_{y'} w(|y - y'|; b) dF(y') \tag{3}$$

This framework has been adopted and further developed by Esteban et al. (2007), who measure identification as in Equation (3) and alienation as the average continuous distance

[2]For example, if $y = 2,000$ and $b = 200$, all individuals with an income between 1,800 and 2,200 are part of y's group.

outside the identification window

$$A(y;F) = \int_{y'} \max(|y-y'|-b,0)dF(y') \tag{4}$$

Using Equation (3) and (4), total polarization for continuously measured incomes is defined as

$$P^{\text{ERG07}}(\alpha,F) = \int_y \left(\int_{y'} w(|y-y'|;b)dF(y')\right)^{\alpha} \left(\int_{y'} \max(|y-y'|-b,0)dF(y')\right) dF(y). \tag{5}$$

Equation (5), therefore, provides a continuous version of ER's original discrete polarization index. Note that Esteban et al. (2007) model identification at income level y as the continuous sum of the weights inside the identification window defined around y, and allow alienation only *outside* the identification window, measuring income distances for alienation from the boundary b. Alternatively, Duclos et al. (2004) proposed a continuous index of the form

$$P^{\text{DER04}}(\alpha,f) = \int_y \left(\int_{y'} f(y)^{1+\alpha} f(y')|y-y'|dy'\right) dy \tag{6}$$

where identification experienced at income y is given by $f(y)^{\alpha}$ and alienation between individuals of incomes y and y' is given by $|y-y'|$. Averaging the product of identification and alienation over y and y' leads to (6). Note that Duclos et al. (2004) make no explicit reference to an identification window (and derive (6) from a completely different axiomatic foundation). However, an implicit identification window is indirectly introduced in the model by the estimation of f through kernel density estimation methods implemented in Esteban et al. (2007). With $f(y)$ empirically estimated by the kernel density estimator

$$\hat{f}(y) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{h} K\left(\frac{z_i - y}{h}\right)$$

the kernel function K effectively plays the role of $w(;b)$, and the kernel bandwidth h corresponds to the identification window size b.

In spite of differences in the derivation of the two indices, the key difference between $P^{\text{ERG07}}(\alpha,F)$ and $P^{\text{DER04}}(\alpha,f)$ is merely that while the former rules out alienation between any two individuals within a common identification window, the latter allows for both identification *and* alienation to be simultaneously felt between two individuals holding different incomes, albeit with identification declining in income distance and alienation increasing in income distance. The latter is therefore 'smoother' – since no discontinuity needs to be introduced at the boundaries of identification windows – and will be the starting point of our inter-temporal measure.

3 Inter-temporal income polarization

One limitation of ER's measurement model is the neglect of time. At the core of the model is the idea that two individuals with similar (resp. different) income feel identified (resp. alienated) to one another. Individual incomes, however, notoriously change over time. Two persons with similar income at time T—when polarization is measured—are likely to have had different incomes in the past and may have ended up around that common income 'pole' from different experiences – say, a large income drop for one and a large income rise for the other. It is natural to consider that these two individuals will not feel equally alienated or identified to one another as if they have had similar incomes for a longer time period. The degree of polarization is therefore affected by the dynamics of incomes in periods prior to T. To put it differently, given two societies with an identical distribution of income at time T, the society with higher income mobility can be expected to be less polarized.

To address this concern, we propose an extension of ER's measure of polarization incorporating a notion of inter-temporal (historical) distance in the formation of effective antagonism and its constituent notions of identification and alienation. Our conceptualization of polarization over several periods is the concentration around poles of income trajectories, rather than of income values in one period.

ER's polarization measure is the sum of effective antagonisms between all pairs of individuals in the society. Effective antagonism of a person earning y towards a person earning y' is itself a function (i) of the alienation felt vis-à-vis each other, depending on the income distance between y and y', and (ii) the strength of identification felt by y towards her own income group. Our proposed inter-temporal polarization measure starts from the same premises: polarization is the sum of effective pairwise antagonisms. However, we allow the two components of effective antagonism—alienation and identification—to depend not only on current incomes, but also on *the history of income differences* between y and y'.

3.1 Inter-temporal pair-wise income distances

We postulate that the alienation between y and y' and the contribution of y' to y's feeling of identification depend on a combination of current and past income differences between the two individuals. More precisely, given two income trajectories over T periods (leading up to current income T) $\mathbf{y} = (y_1, y_2, ..., y_T)$ and $\mathbf{y'} = (y'_1, y'_2, ..., y'_T)$, we consider a measure of inter-temporal income distance $D(\mathbf{y}, \mathbf{y'})$ between the two satisfying the properties of Monotonicity, Independence, Relevance and Normalization defined as follows.

Property 1 (Monotonicity)**.** *For any three income vectors,* $\mathbf{y} = (y_1, y_2, ..., y_T)$, $\mathbf{y'} = (y'_1, y'_2, ..., y'_T)$ *and* $\mathbf{y''} = (y''_1, y''_2, ..., y''_T)$ *such that* $y'_t = y''_t$ *for all* $t \neq s$ *and* $|y_s - y''_s| \geq |y_s - y'_s|$, $D(\mathbf{y}, \mathbf{y''}) \geq D(\mathbf{y}, \mathbf{y'})$

Monotonicity means that the inter-temporal distance cannot decrease with an increase

in any cross-section distance $|y_s - y'_s|$, leaving all other cross-section distances unchanged.

Property 2 (Independence)**.** $\forall\ s,t \in 1,2,...,T,\ \frac{\partial^2 D(\mathbf{y},\mathbf{y'})}{\partial|y_t - y'_t|\partial|y_s - y'_s|} = 0.$

Independence implies that in the aggregation process of period distances, the effect on the inter-temporal distance $D(\mathbf{y},\mathbf{y'})$ of each period distance $|y_t - y'_t|$ is independent of any other period distance and does not depend on the level of $|y_t - y'_t|$ in any given period.

Property 3 (Relevance)**.** $\forall\ t \in 1,2,...,T$*,if* $|y_t - y'_t| = 0$*, then the contribution of* $|y_t - y'_t|$ *to* $D(\mathbf{y},\mathbf{y'})$ *is nil.*

Relevance requires the current distance $|y_t - y'_t|$ at any period t to be positive to contribute to the inter-temporal distance.

Property 4 (Normalization)**.** *If* $|y_1 - y'_1| = |y_2 - y'_2| = ... = |y_t - y'_t| = |y_T - y'_T|$*, then* $D(\mathbf{y},\mathbf{y'}) = |y_t - y'_t|$.

Given Normalization, if the period distances are all equal, then the inter-temporal distance $D(\mathbf{y},\mathbf{y'})$ coincides with them. This property ensures that the order of magnitude of the inter-temporal distance is the same as the one of single-period distances.

The two properties of Monotonicity and Independence imply that the inter-temporal distance $D(\mathbf{y},\mathbf{y'})$ has an additive functional form. Independence and Relevance require the inter-temporal distance to be linear in each period distance. This result is expressed in Proposition 1 (a proof is provided in Appendix A).

Proposition 1. *The inter-temporal distance* $D(\mathbf{y},\mathbf{y'})$ *satisfies Properties 1, 2, 3 and 4 for any sequence of income distances* $\{d_t = |y_t - y'_t|\}_{t=1}^T$ *if and only if*

$$D(\mathbf{y},\mathbf{y'}) = \sum_{t=1}^{T} \phi_t\, |y_t - y'_t| \tag{7}$$

with $\{\phi_t : \phi_t \geq 0\}_{t=1}^T$ *a sequence of non-negative constants such that* $\sum_{t=1}^T \phi_t = 1$.

The sequence of weights $\{\phi_t : \phi_t \geq 0\}_{t=1}^T$ reflects the relative importance of current and historical income differences in forming current (period T) feelings of alienation and identification. We discuss some desirable properties for $\{\phi_t\}_{t=1}^T$ shortly, but make two remarks first.

One, we will allow identification and alienation to be governed by different sequences of weights (and therefore different inter-temporal income distances). As we discuss below, this allows for distinct roles of past income differences in building up identification and generating alienation. For clarity, we do not introduce distinct notation at this stage.

Two, time-additivity in (absolute) income differences is consistent with a myopic cumulative absolute difference model: contemporaneous income gaps between y and y' accumulate in inter-temporal differences, independently of income differences in any other time period. At each period, agents y and y' compare their incomes and the distance accumulates over time. To be clear, this rules out inter-temporal compensation in which

income differences $y_s > y'_s$ in period s could be partially 'compensated' in period t when $y_t < y'_t$. Therefore, there is no consideration of inter-temporal income smoothing in forming alienation and identification. In this respect, our approach differs from using the distance between permanent incomes as a proxy for inter-temporal distance.

Moving on, we now discuss a number of desirable restrictions on the sequence $\{\phi_t\}_{t=1}^{T}$. As pointed out by Calvo and Dercon (2009) in the closely-related context of chronic poverty measurement, the choice of an inter-temporal aggregation rule requires some judgements on the relative importance of the present with respect to the past. For example, a simple unweighted sum implicitly gives the same weight to present and past periods. In our approach, the reference period for comparing incomes is the present, but past values can have relevance today through a memory process. People can have some memory of their income in previous periods, but not necessarily a full one: it is reasonable to assume that the further back in time we go, the fewer past differences matter today.

We formalize this memory process through a set of properties for the sequence of weights $\{\phi_t\}_{t=1}^{T}$.

Property 5 (Memory Erosion)**.** $\forall s, t : 1 \leq s < t \leq T$*,* $\phi_s \leq \phi_t$.

Memory Erosion describes a model in which the more distant in the past income differences are observed, the lower is their contribution to the inter-temporal distance measure. This reflects a weakly monotonic decay in the recall (or memory) of past income gaps.[3]

Property 6 (Bounded Memory)**.** $\exists\, 1 \leq t_0 \leq T s.t.\ \forall s < t_0$*,* $\phi_s = 0$.

Bounded Memory further assumes that there exists a point in the past beyond which income distances do *not* contribute anymore to *current* inter-temporal distance measure. Bounded Memory is a technical property without which inter-temporal distance measures cannot be empirically estimated.

The two properties of Normalisation and Memory Erosion together lead to the following lemma of Cross-Section Consistency which guarantees that if agents are assumed to 'live' only one year (or, equivalently, if they are memory-less so that $t_0 = T$), the inter-temporal distance boils down to ER's cross-sectional income distance.

Lemma 1 (Cross-section Consistency)**.** $t_0 = T \iff \phi_T = 1$

Property 7 (Convexity in time)**.** $\forall\ s < t$*,* $k > 0$*,* $(\phi_s - \phi_{s-k}) \leq (\phi_t - \phi_{t-k})$

Convexity in time states that $\{\phi_t\}_{t=1}^{T}$ is taken to be a convex sequence: the marginal contribution of time t income distances to the inter-temporal distance declines with its distance to the current period at a declining speed. Convexity implies that inter-temporal shifts back in cross-sectional income distances (i.e., shifts that make incomes closer at time t but more

[3] Memory Erosion rules out, for example, a model of 'impressionable years' in which income differences in particular young ages have a stronger impact than adulthood income differences, or a model in which feelings of alienation and identification are cemented during childhood years. Such a model would however also be inconsistent with the cross-section polarization measurement of Esteban and Ray (1994).

distant at time $t-1$) have higher impacts on inter-temporal income distance, the closer the shift is to T. Technically, consider the vector of income distances $\mathbf{d} = \{|y_t - y'_t|\}_{t=1}^{T}$ and let $\boldsymbol{\delta}(t)$ be a vector that shifts a small income distance δ from time t to time $t-1$.[4] Then, if we define $\mathbf{d}_t = \mathbf{d} + \boldsymbol{\delta}(t)$, Convexity in time means that $\mathbf{d}_t \geq \mathbf{d}_s \; \forall t > s$.

Compound discounting A convenient sequence of weights satisfying properties (1)-(6) can be derived through a continuous compound discounting process: $\phi = \left\{ \frac{e^{-rt}}{\sum_{t=0}^{T-1} e^{-rt}} \right\}_{t=1}^{T}$, where r is a non-negative discount rate. The inter-temporal income distance is therefore modelled as

$$D(\mathbf{y}, \mathbf{y'}; r) = \frac{\sum_{s=0}^{T-1} e^{-rs} |y_{T-s} - y'_{T-s}|}{\sum_{s=0}^{T-1} e^{-rs}} \tag{8}$$

such that $D : 2 \times T \mapsto R$ takes the two income vectors **y** and **y'** as input, and outputs a scalar summarizing the inter-temporal distance between the two.

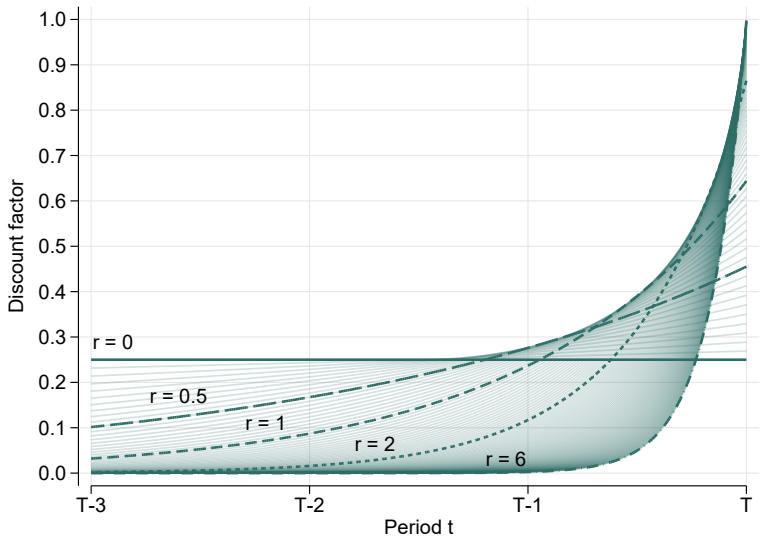
The memory process For a given parameter r in Equation (8), the weight is decreasing going backwards in time unless $r = 0$. Since the reference period for the inter-temporal distance is the last one (T), the discount rate r is a parameter of inverse *memory*: the larger the discount rate, the faster income differences distant in the past lose their importance today.

Figure 1 helps visualise the discounting process plotting the sequence of discounting factors $\phi_t = (e^{-rt})/(\sum_{t=0}^{T-1} e^{-rt})$ as a function of time t for an example situation of four periods. If there is maximum memory ($r = 0$), then all the periods have the same weight $1/T = 0.25$, and the inter-temporal distance coincides with the simple average period distance. In all the other cases, the weight of the present period outweighs those of the past ones, and the speed of convergence to zero while going backwards in time depends on the value of the parameter r: the higher the discount rate r, the faster the convergence to zero. If there is no memory at all, a discount rate of $r = 6$ is enough to set to (practically) zero all the weights but the one for the present (0.00247 for $t = T - 1$ –one 400th of the weight at T –, 0.00000613 for $t = T - 2$ – one 163,000th of the weight at T – and so on). Intermediate values of r between zero and six allow setting different degrees of memory.

Figure 2 provides a further illustration of how the inter-temporal distance behaves depending on the strength of memory and on the correlation in time between the two trajectories **y** and **y**′. We see in the figure two examples of income vectors in a time frame of three periods, one in which the two income trajectories are diverging over time (upper-right panel) and one in which they are converging (lower-right panel). When r is zero (maximum memory), the inter-temporal distance coincides with the average of the period distances $|y_t - y'_t|$ since every period has the same weight $1/3$. As r increases, and the memory fades, the inter-temporal distance is closer and closer to the present one. When the two income trajectories are diverging, so that the distance in the last period is larger

[4] For example, $\boldsymbol{\delta}(2) = (+\delta, -\delta, 0, 0, ..., 0)$ $\boldsymbol{\delta}(3) = (0, +\delta, -\delta, 0, ..., 0)$, etc.

Figure 1: Visualising the discounting process



Note: The figure plots the discount factor $\phi = (e^{-rt})/(\sum_{t=0}^{T-1} e^{-rt})$ as a function of time for four periods, with T being the present. The lines are drawn using different values of the parameter of memory r, going from 0 (maximum memory) to 6 (minimum memory) by steps of 0.05.

than the average one, the less there is memory of past closeness, the more the two trajectories are distant in a temporal perspective. On the contrary, if the two income vectors are converging, so that the distance today is lower than the distance in the past, then less memory translates into smaller inter-temporal distance.

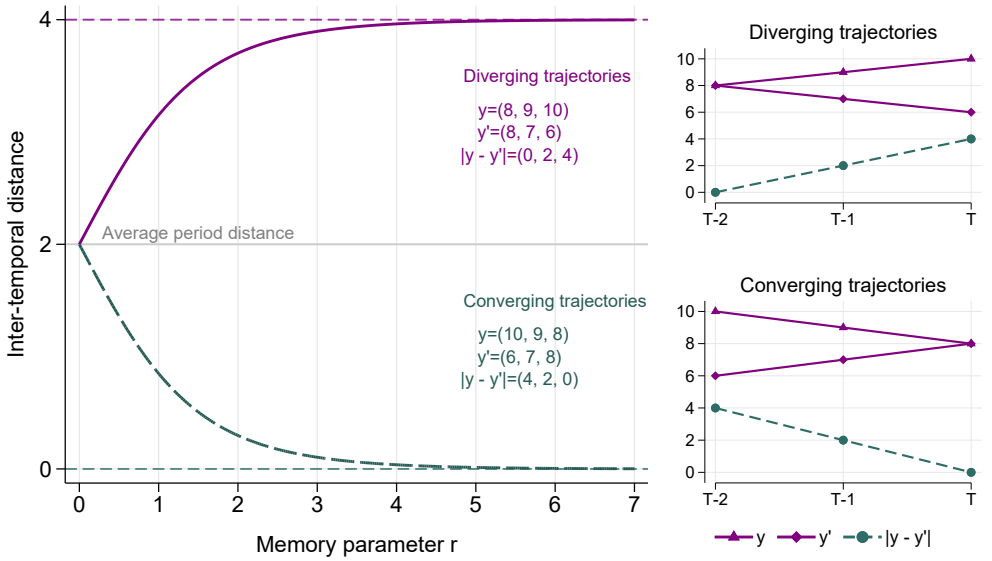
3.2 Inter-temporal income polarization

Endowed with a measure for the pairwise distance between income trajectories evaluated at contemporaneous time T, we apply the identification-alienation framework as in the cross-sectional setting described in Section 2: we only change the notion of distance on which group membership and individual alienation are based.

Differently from the approach of Esteban et al. (2007) – and in the spirit of Duclos et al. (2004) instead – we allow alienation also inside the identification window to avoid the discontinuity at the boundaries and to make the alienation component of the index fully comparable with the Gini inequality index. Remaining in the cross-sectional context for a moment, while we take the definition of identification from Equation (3), alienation at y is the continuous sum of the absolute income differences with respect to the rest of the income values y':

$$A(y;F) = \int_{y'} |y - y'| dF(y') \tag{9}$$

Figure 2: The impact of memory on the inter-temporal distance



Note: On the right-hand side, the figure shows two income vectors **y** and **y'** and their period-by-period distance, separating the cases of diverging and converging trajectories. On the left-hand side, the figure plots the inter-temporal distance D (see Equation 8) between the two income vectors as a function of the parameter of memory r. Consider that higher r means weaker memory. The two lines represent the pattern of D when the two trajectories are diverging (solid line) and converging (dashed line).

Therefore, income polarization is defined as:

$$P(F;b,\alpha)=\int_y\left[\left(\int_{y'}w(|y-y'|;b)\right)^{\alpha}\int_{y'}|y-y'|dF(y')\right]dF(y) \tag{10}$$

Our strategy to incorporate the longitudinal dimension in the definition of polarization from Equation (10) is simply replacing the cross-sectional distance $|y-y'|$ with the inter-temporal one $D(\mathbf{y},\mathbf{y'};r)$ defined in Equation (8).

As shown in Equation (10), income distances are relevant for both the identification and the alienation components of antagonism: the absolute distance determines whether or not two income values belong to the same group (depending on $|y-y'|\gtreqless b$), and it is also directly the value of reciprocal alienation. We allow the discount factors' sequence $\{\phi\}_{t=1}^{T}$ to differ for the two components: we call r_I the discount rate capturing the memory for identification, and r_A that for alienation. If $r_I > r_A$, the past experience is less important—more discounted—when it comes to deciding who belongs to one's own group, rather than in determining reciprocal alienation. On the contrary, if $r_I < r_A$ the alienation component is more focused on present differences, while the proximity perceptions are more sensitive to past closeness. Of course, they may also coincide, and the choice depends on the preferred behavioural model.

Starting from identification, let $g(\mathbf{y})$ be the continuous multivariate density of income trajectories; then, the extent of identification for the income trajectory **y** is given by the

continuous sum of the weights over all the other income vectors **y'**:

$$I(\mathbf{y}, G; b, r_I) = \int_{\mathbf{y'}} w(D(\mathbf{y}, \mathbf{y'}; r_I); b) dG(\mathbf{y'}) \tag{11}$$

Equation (11) gives a scalar summarizing the inter-temporal identification for the income vector **y**, given the degree of memory r_I, the identification window width b around **y**, and the weighting scheme w that assigns a positive weight to income trajectories inside b, and a weight of zero outside.

For the alienation component, we add the time dimension by computing pairwise income differences through the inter-temporal distance $D(\mathbf{x}, \mathbf{y}; r_A)$:

$$A(\mathbf{y}, G; r_A) = \int_{\mathbf{y'}} D(\mathbf{y}, \mathbf{y'}; r_A) dG(\mathbf{y'}) \tag{12}$$

The final measure of inter-temporal polarization is the average over all income trajectories of the effective antagonisms towards the rest of the trajectory distribution, in analogy with the cross-sectional case. For each income trajectory **y**, the inter-temporal effective antagonism is defined as the product between identification to the power of α and alienation:

$$EA(\mathbf{y}, G; b, r_I, r_A, \alpha) = I(\mathbf{y}, G; b, r_I)^{\alpha} A(\mathbf{y}, G; r_A) \tag{13}$$

The resulting inter-temporal polarization is the average antagonism in the distribution of income trajectories, defined in Equation (14). To ease comparability between indices computed for different values of α, we rescale the index by raising it to the power of $1/\alpha$.

$$IP(G; b, r_I, r_A, \alpha) = \left[\int_{\mathbf{y}} \left(\int_{\mathbf{y'}} w(D(\mathbf{y}, \mathbf{y'}; r_A); b) dG(\mathbf{y'}) \right)^{\alpha} \left(\int_{\mathbf{y'}} D(\mathbf{y}, \mathbf{y'}; r_A) dG(\mathbf{y'}) \right) dG(\mathbf{y}) \right]^{\frac{1}{\alpha}} \tag{14}$$

To make the index operational, one needs to choose a domain for b and a weighting function inside it. We will discuss some of the choices for empirical analysis in the part of the paper devoted to the estimation (Section 4), and in the application (Section 5).

4 Estimation

4.1 Calculation

Assuming a sample of n individual income vectors $\{\mathbf{y}_i = y_{i1}, y_{i2}, ..., y_{iT}\}_{i=1}^{N}$, the estimation of the inter-temporal polarization index in Equation (14) aggregates the effective antagonism of each person in the sample. For each trajectory $\mathbf{y}_i$ in the data, we compute the pairwise inter-temporal distance with respect to all the other trajectories $\mathbf{y}_j$ in the sample as follows:

$$\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r) = \frac{\sum_{s=0}^{T-1} e^{-rs} |y_{i,T-s} - y_{j,T-s}|}{\sum_{s=0}^{T-1} e^{-rs}} \tag{15}$$

Since the distance $\hat{D}$ is symmetric ($\hat{D}_{ij} = \hat{D}_{ji}$), we end up with $\frac{n(n-1)}{2}$ inter-temporal distances. If the discount rate for alienation is different from that of identification, the distance in Equation (15) is computed twice: once using r_A and once using r_I, obtaining $n(n-1)$ values of distances. We distinguish the two distances using the notation $\hat{D}^I_{ij}$ for $\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r_I)$ and $\hat{D}^A_{ij}$ for $\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r_A)$.

The individual identification component in Equation (11) is estimated by using the sum of the weights of people belonging to one's group, where the group is defined as those falling inside one's identification window. Let b_i be the identification window width for person i and $w_{ij}(\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r_I); b_i)$ be the weight attached to person j depending on the inter-temporal distance between the two $\hat{D}_{ij}$ and on the identification window width of i. If $\hat{D}_{ij}$ falls inside b_i, the weight w_{ij} is positive; outside, it is zero. We discuss in Section 4.2 the details and implications of the weighting process.

$$\hat{I}_i(\mathbf{y}_i; b_i, r_I) = \frac{\sum_j w_{ij}(\hat{D}^I_{ij}; b_i)}{n} \tag{16}$$

The alienation component is computed as the average distance between the income trajectory of i and that of any other person in the population:

$$\hat{A}_i(\mathbf{y}_i; r_A) = \frac{\sum_j \hat{D}^A_{ij}}{n} \tag{17}$$

For each person i, the effective antagonism towards the rest of the population is given by the product of her identification to the power of α, and her average alienation from the rest of the income trajectories:

$$\hat{EA}_i(\mathbf{y}_i; \alpha, r_I, r_A, b_i) = \left[\hat{I}_i(\mathbf{y}_i; b_i, r_I)\right]^{\alpha} \hat{A}_i(\mathbf{y}_i; r_A) \tag{18}$$

Combining Equations (16), (17), and (18), the final estimate of inter-temporal polarization in Equation (14) is the average antagonism in the sample:

$$\hat{IP}(\alpha, r_I, r_A, b_i) = \left[\frac{\sum_i \sum_j \left(\sum_j w_{ij}(\hat{D}^I_{ij}; b_i)\right)^{\alpha} \hat{D}^A_{ij})}{n^{\alpha+2}}\right]^{\frac{1}{\alpha}} \tag{19}$$

The formulas above can be modified to include sample weights, and we can obtain homogeneity of degree zero (scale invariance) by mean-normalizing income in each period.

Finally, since the index in Equation (19) is an average, we can aggregate the antagonism at the individual level by subgroups, as long as they do not overlap and represent a complete partition of the population:

$$\hat{IP} = \left(\sum_{k=1}^{K} \hat{IP}_k\right)^{\frac{1}{\alpha}} = \left(\sum_{k=1}^{K} \frac{\sum_{i \in n_k} \hat{EA}_i}{n_k}\right)^{\frac{1}{\alpha}}, \qquad \sum_{k=1}^{K} n_k = n \tag{20}$$

Notice that the individual effective antagonism $\hat{EA}_i$ is computed as in Equation (18), thus using the whole population as the reference for identification and alienation. This implies that $I\hat{P}_k$ conveys different information from $I\hat{P}$ computed on the subset k of the population: the index computed separately for each subgroup informs on the level of inter-temporal polarization *within* each subgroup regardless the rest of the population; on the other hand, the element $I\hat{P}_k$ measures the average antagonism for group k with respect to the whole population. This decomposition allows us to compute how much of the overall antagonism can be attributed to different subgroups – of gender, level of education, occupation, geographic area etc. –, and therefore to identify 'hot spots' of potential conflict.

4.2 Identification weights

The width of the identification window b can be set in several ways: we group them into two main categories. One possibility is a *relative* threshold, function of one's own income, that makes the identification window width increase linearly with income. Another possibility is an *absolute* threshold fixing the identification window width at the same value for every person in the distribution. A relative threshold approach is consistent with traditional inequality and polarization analysis based on relative differences. In the inter-temporal context, the identification window should be defined around an inter-temporal notion of income: let $\hat{Y}_i$ be the average income weighted using the discount rate for identification r_I:

$$\hat{Y}_i(\mathbf{y}_i; r_I) = \frac{\sum_{s=0}^{T} e^{-r_I s} y_{i,s}}{\sum_{s=0}^{T} e^{-r_I s}} \tag{21}$$

Then, the identification window width for person i is defined as a fraction p of her inter-temporal income: $b = p\hat{Y}_i$.

Along with the identification window, the choice for the weighting scheme is to be made The weight must be defined for each pair of income trajectories: w_{ij} is the weight of income trajectory $\mathbf{y}_j$ for income trajectory $\mathbf{y}_i$, and is a function of their symmetric distance D_{ij}, and of the identification window of $\mathbf{y}_i$. We set $w(0) = 1$ so that the maximum weight goes to people having exactly the same income trajectory, and the identification group is never empty (at least self-identification is true). Moreover, the function is chosen so that $w(\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r) \geq b) = 0$, where b is the maximum distance allowed for identification: outside the identification window the weight is always zero.

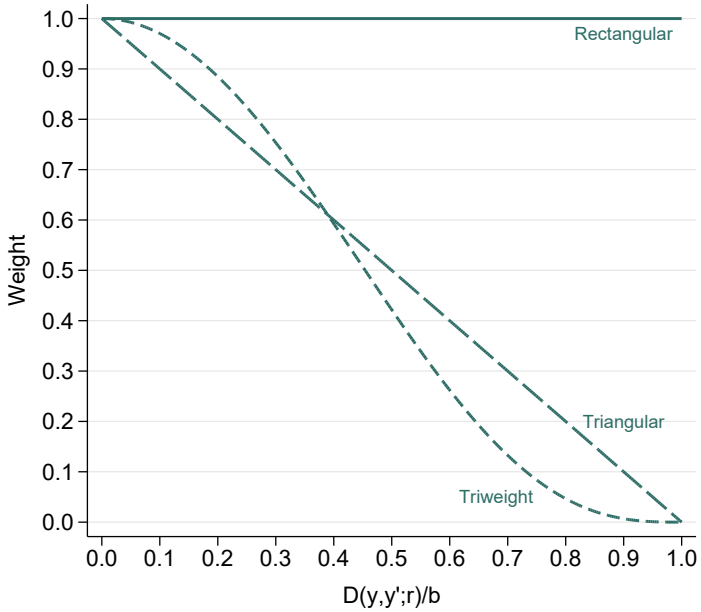
We propose in Equation (22) three possible weighting functions coherent with the con-

ditions $w(0) = 1$, and $w(\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r) \geq b) = 0$ and plot them in Figure 3.

$$\begin{cases} w_{ij}^{rectangular}(\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r)) = 1 & \text{if } |y_j - y_i| \leq b_i \\ w_{ij}^{rectangular}(\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r)) = 0 & \text{otherwise} \\ \\ w_{ij}^{triangular}(\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r)) = max[0, 1 - \frac{\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r)}{b_i}] \\ \\ w_{ij}^{triweight}(\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r)) = max[0, (1 - (\frac{\hat{D}_{ij}(\mathbf{y}_i, \mathbf{y}_j; r)}{b_i})^2)^3] \end{cases} \tag{22}$$

Rectangular weights is reminiscent of Esteban and Ray (1994)'s original model with discrete income classes, but is the least attractive option with continuously measured incomes: it implies that the feeling of identification is not a function of distance within a window, and leads to a discontinuity at the boundary of the window. It is, however, the easiest choice in terms of interpretation: since each person counts for one unit, the identification component is simply the proportion of the population falling inside the identification window.

Figure 3: Possible weighting functions



Note: The figure plots three possible weighting schemes based on Kernel functions. They share the properties of assuming the maximum value of one when $D(\mathbf{y}, \mathbf{y'}; r) = 0$, and the minimum value of zero when $D(\mathbf{y}, \mathbf{y'}; r) \geq b$.

On the contrary, when using the triangular or the triweight schemes, the inclusion into the identification group is smoother: more weight is given to income trajectories close to one's own, while for the others inside the identification window the weight falls when approaching the boundary. Moreover, while the triangular weights imply a linear decay, the triweight scheme gives more importance (as compared to the triangular) to observations

below a certain value of the $\frac{\hat{D}_{ij}(\mathbf{y}_i,\mathbf{y}_j;r)}{b_i}$ ratio, and less to observations above it. Figure 3 illustrates this clearly. Of course, other weighting schemes are possible and those in Equations (22) are not exhaustive.

5 Inter-temporal polarization across cohorts of Italian workers

We provide in this section an application of the index developed in Section 3 using matched survey-administrative data for Italy. The main goal of this application is to detect whether changing the relevance of past differences for identification and alienation r_I and r_A captures different aspects of the polarization process. We compute separately for successive cohorts of Italian workers the level of inter-temporal polarization over ten years of earnings. A cohort analysis means that the reference group for identification and alienation sentiments are those having a similar age in the same years. We examine whether the measured levels and patterns of polarization are affected by varying the parameters of memory. We also show a heterogeneity analysis by gender, level of education, and geographic area of work, which we suspect to be important correlates of the polarization process.

5.1 Data

Data source The inter-temporal index of polarization requires income trajectories as input, so we need panel data that follow people over time. The pairwise comparisons of income trajectories in Equation (8) also require a balanced panel – that every person is observed in the same periods. The data used for the application are a subset of the Administrative-SILC (AD-SILC) panel dataset, developed by merging through fiscal codes the waves from 2004 to 2017 of the IT-SILC survey (the Italian component of the European Union Statistics on Income and Living Conditions, EU-SILC) with social security records collected by the Italian National Social Security Institute (INPS). The INPS archives record employment and earnings histories of all individuals working in Italy, collecting demographic characteristics, gross annual earnings, allowances, weeks worked in the year, and the type of employment contract. The EU-SILC component allows us to exploit individual-level information usually not available in administrative data, as the highest level of education.

Sample selection The sample is restricted by excluding individuals without Italian citizenship, under-represented in older cohorts. We focus on employees in the private sector, the only category covering a very long-time span in the INPS archives (from 1974 on). However, our measure of economic well-being — real annual earnings at 2015 price level, gross of personal income taxes and social contributions — includes individual income from any job, also from atypical work and self-employment, and allowances for sickness, ma-

ternity and CIG.[5] We follow workers born between 1940 and 1973 for the eleven years in which they are aged 35–45, with at least six years of positive earnings. Periods of non-employment observed in the data are counted as zero income. In terms of calendar years, we observe earnings patterns from 1975 to 2018. The bottom and top 0.1% of the earnings distribution in each year are dropped to minimize measurement errors at the tails and get rid of severe outliers. Earnings are mean-normalized within the cohort and age to ensure the scale invariance of the index and therefore its comparability over time.

The workers included are divided into thirty five-year rolling cohorts of birth, each of which overlaps with the preceding one for every year but the last one. The final sample includes 26,645 workers, of which 16,720 men and 9,925 women, divided into cohorts of birth from 1940–1944 to 1969–1973. Summary statistics on the selected sample are available in Table B.1 and Table B.2 in the Appendix.

5.2 Choice of parameters

The inter-temporal polarization index is computed separately for each cohort of birth using IT-SILC sample weights. We repeat the computation of the inter-temporal polarization index for $\alpha = 1$ — minimum sensitivity to identification — and for $\alpha = 1.6$ — maximum sensitivity. When setting the degree of memory, we use all the combinations of $r_A, r_I = 0, 0.5, 6$. If the discount rate is high ($r_I, r_A = 6$) only the value of income in the last period — at age 45 — has a positive weight, and the past is completely forgotten. If $r_I, r_A = 0.5$, the discount process is such that the closer the age approaches 45 the more weight the income gains, but income in the first period — at age 35 — has a weight of zero. Finally, $r_I, r_A = 0$ means no discount of the past, so that every age has the same weight equal to $1/11$.

In the baseline specification, we use all the possible combinations of $\alpha = 1$ and $\alpha = 1.6$ with $r_I, r_A = 0, 0.5, 6$, an identification window of 20% of own inter-temporal income, and a triweight Kernel to weight the neighbours inside one's identification window.[6] We provide in the Appendix also results with triangular and rectangular weights, and the results using an absolute threshold for the identification window defined as 20% of average within-cohort inter-temporal income.

Only for the specifications with $r_I = r_A$ for brevity, we report in the Appendix a plot of the indices with normal-based confidence intervals (Figure B.5) and a table with the standard errors (Table B.3) computed through 1,000 bayesian bootstrap repetitions.

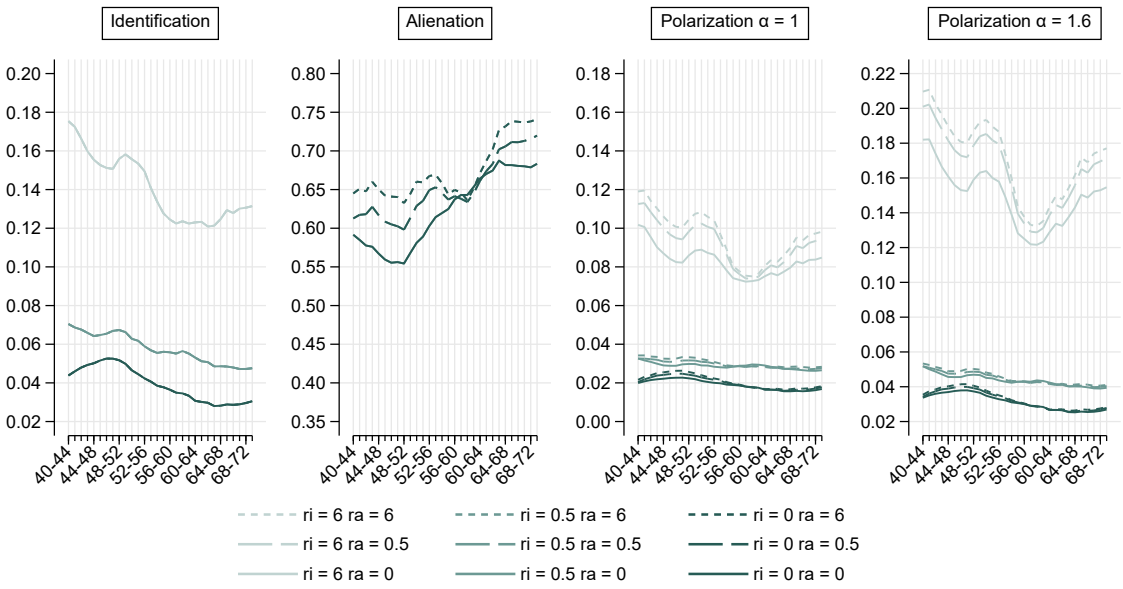
[5] The Cassa Integrazione Guadagni (CIG) is an Italian short-time work scheme for supporting the wages of employees of firms going through crisis events. It is limited in time and subject to specific requirements for both the employer's nature and situation and the employment contract.

[6] Since we include observations with zero income, setting a relative threshold mechanically forces the identification window for zero earners to be zero. This implies that identification for zero-income earners is only possible with other zero-income earners, which is consistent with the identification of those not working with the group of non-employed. Results with only positive earnings are reported in the Appendix.

5.3 Results

Figure 4 shows the patterns across cohorts of inter-temporal identification, alienation and polarization for the minimum ($\alpha = 1$) and the maximum ($\alpha = 1.6$) value of sensitivity to identification, and for all the combinations of the parameters of memory $r_I, r_A = 0, 0.5, 6$. To read the graph, consider that a darker colour indicates stronger memory for identification, while a more solid pattern indicates stronger memory for alienation. The identification component is not sensitive to the discount rate for alienation r_A; therefore, only the three lines corresponding to the three values of r_I are plotted on the left-hand panel of Figure 4. The same applies to the alienation component, which does not depend on the discount rate r_I but only on r_A. The degree of memory seems to be relevant for identification in terms of both level and pattern: in case of no memory ($r_I = 6$), the average proportion of people inside one's identification group at age 45 is between 12 and 17.5% depending on the cohort. If $r_I = 0.5$—the memory decays going backwards in time reaching zero at age 35—the average fraction of neighbours falls between 4.7 and 7%. Increasing memory to the maximum ($r_I = 0$), the identification is between 2.8 and 5.3%. This suggests that, in the process of including nearby people in income groups, which is the basis of the identification process, incorporating past income differences leads to a lower rate of inclusion, distancing from each other people who would be closer if only income at age 45 were taken into account.

Figure 4: Inter-temporal polarization and its components by cohort



Note: The figure plots the value of average inter-temporal identification, alienation and polarization of gross real annual earnings for all the possible combinations of three values $(0, 0.5, 6)$ of the discount rates of past differences for identification r_I and for alienation r_A. The identification window width is defined as 20% of own inter-temporal earnings, and an adjusted triweight weighting scheme is applied to weight observations inside one's identification window. The sample includes employees in the private sector, and the indices are computed separately for thirty five-year rolling cohorts of birth, from 1940-1944 to 1969-1973 among workers observed every year from age 35 to 45. Workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

The inter-temporal identification has a clear long-run decreasing trend, falling by 25.1% with minimum memory, 30.4% with maximum memory, with a peak around cohort 1946–1950, falling thereafter, and a recovery for younger cohorts from 1964-1968 on. The trends are comparable when changing the memory parameter r, but relevant differences emerge for the oldest cohorts: when focusing only on present income, inter-temporal identification falls by 14.1% from cohort 1940-1944 to cohort 1947-1951; on the other extreme, if the past is weighted as the present, we see an increase in identification of 19.9%. This means that, for those cohorts, people were moving from bigger to smaller sized groups in terms of earnings at age 45, reducing cross-sectional group identity in middle-career, while they were moving from smaller to bigger groups in terms of income trajectories, increasing group identity in terms of career paths.

A possible explanation for this pattern relies on the functioning of the Scala Mobile – 'elevator' – wage indexation mechanism adopted in Italy from the 1970s to the early 1990s (weakened from 1984 before final abolition in 1992). It was designed for granting the same absolute wage increase to all employees in a period of sustained inflation, inducing mechanically greater proportional wage changes at the bottom of the distribution (Manacorda, 2004). The indexation may have played a role in making income trajectories more similar than before for workers affected by the Scala Mobile. Cohorts 1940–1944 to 1945–1949 in our analysis were completely covered by the mechanism during age 35–45; later on, falling inflation and the reforms aimed at reducing the wage increases made the equalizing power increasingly weak.

Moving to inter-temporal alienation, we can appreciate in the second panel from the left in Figure 4 that the level and the upward trend of inequality are comparable to available estimates of the Gini index for Italy in the private sector.[7] We see that increasing the memory for alienation ($\downarrow r_A$, more solid pattern in the figure) lowers the level of inequality for almost every cohort: the more people remember their past income, the less alienation they perceive, meaning that incorporating past experience reduces long distances between people. This behaviour is consistent with a world of mostly diverging trajectories: when the income distances today are larger than those in the past, including past experience makes the inter-temporal distance decrease. Interestingly, this mechanism is not in place for a bunch of cohorts around cohort 1953-1957 for which alienation is almost independent of the parameter of memory, uncovering very persistent inequalities.

The two panels on the right in Figure 4 combine inter-temporal identification and alienation as in Equation 19 using two different levels of sensitivity to identification $\alpha = 1, 1.6$. We notice that polarization is influenced more by identification than by alienation; the more the inter-temporal polarization index is sensitive to group identity, the more its pattern resembles that of identification. The overall trend of inter-temporal polarization is decreasing despite rising alienation, suggesting that the effect of falling identification is predominant.

[7] When comparing the alienation component of the polarization index with the Gini index, remember that the former should be divided by 2 to avoid double counting of income differences.

To comment on the impact of the discount process on the polarization index, we focus on the case in which $\alpha = 1$. The impact on inter-temporal polarization of increasing memory follows what we saw for identification and alienation separately: when $\downarrow r_I$—stronger memory for identification—the level of polarization rapidly falls for all cohorts, while when $\downarrow r_A$—stronger memory for alienation—there is again a reduction in polarization but it does not affect heavily the cohorts characterized by very persistent inequalities.

The long-run reduction of polarization is similar in magnitude if we use minimum memory (-17.5%) or maximum memory (-15.6). However, as we have seen for identification, the pattern is different: while polarization in income at 45 for older cohorts was falling, it was actually on the rise in terms of income trajectories, and a similar decoupling emerges for the very last cohorts.

We show in the Appendix in Figure B.5 and Table B.3 that the differences commented here are statistically significant by using bayesian bootstrap inference. Moreover, we provide some sensitivity analyses changing the weighting scheme inside the identification window to triangular (Figure B.1) and rectangular (Figure B.2), changing the way of defining the identification window to an absolute threshold of income (Figure B.4), and removing zero earners (Figure B.3). As expected, a less smooth weighting scheme increases the groups' relative size, but all the main results for inter-temporal polarization are robust to different specifications. When excluding workers with even a single zero when 35-45, we report an expected large drop in the level of alienation and we notice that the impact of the zeros on identification is visible almost exclusively when $r_I = 6$; with no memory, there can be a sizable group including zero earners, while moving to income trajectories the periods spent without earnings are smoothed out and the level of identification is lower. The impact of this mechanism is particularly strong for the last cohorts from cohort 1962-1966 on, for whom there is a strong increase in identification when zero-income earners are included and a slightly decreasing trend when they are excluded.

5.4 Heterogeneity

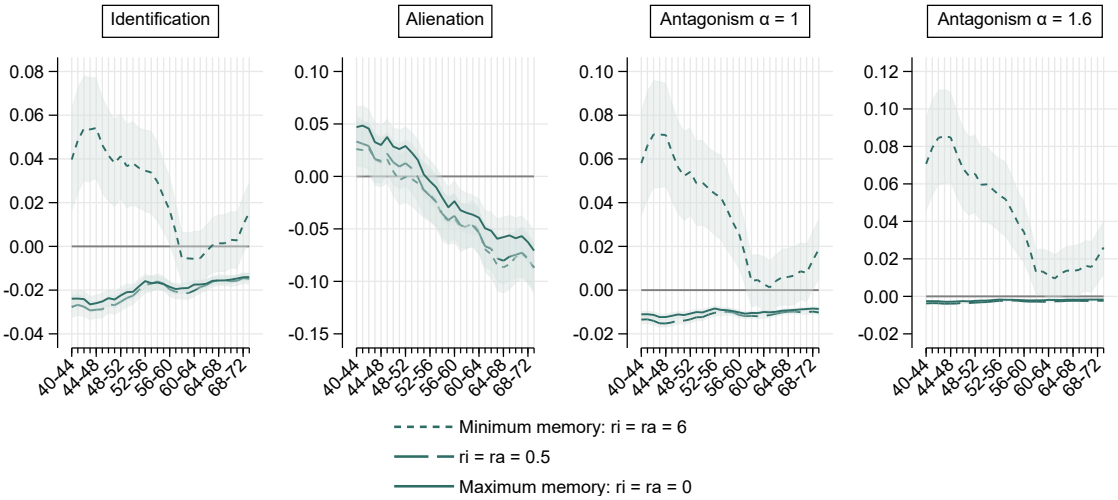
In this section, we explore the heterogeneity in within-cohort inter-temporal polarization and its components by gender, level of education, and geographical area of work. We regress separately for each cohort individual identification, alienation and effective antagonism on indicators for being a woman, tertiary graduate, and working in the South or Islands of Italy, as in Equation (23):

$$z_i = \beta_0 + \beta_1 F_i + \beta_2 G_i + \beta_3 S_i + u_i \tag{23}$$

where z_i can be either identification, alienation, or effective antagonism for person i, F_i is an indicator for whether i is a woman, G_i a tertiary graduate, and S_i a worker in the South or Islands. The proportion of these categories in the sample is available in Table B.1 in the Appendix. On average, but with relevant variation across cohorts and an upward trend

over time, women are 37% of workers, tertiary graduates are 6.9%, and workers in the South and Islands are 23.7%. These regressions exploit the linearity of our index of inter-temporal polarization with respect to individual alienation and identification to assess the contributions of different groups of people to aggregate inter-temporal polarization. We use three specifications changing the level of memory $r = 0, 0.5, 6$, with $r = r_I = r_A$. The identification window is set at 20% of own inter-temporal earnings and the weighting scheme inside the identification window is the triweight Kernel. An alternative specification using only positive earnings for comparison is provided in the Appendix.

Figure 5: Gender differences in inter-temporal polarization



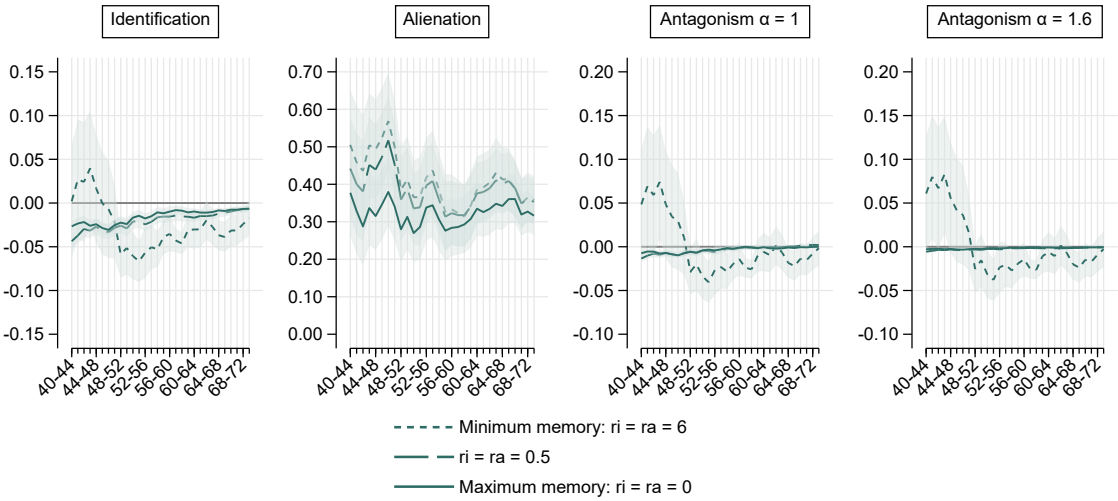
Note: The figure plots the coefficient of the indicator variable for women in a linear regression of inter-temporal identification, alienation, and effective antagonism on three indicator variables for women, tertiary educated, and workers in the South or Islands of Italy. The regressions are performed separately for each cohort of birth and level of the parameter of memory r – 0, 0.5, 6 –, and $r_I = r_A$ to make the results manageable. Consider that higher r means weaker memory. The confidence intervals plotted are at 95% confidence level computed using robust standard errors. The values of individual inter-temporal identification, alienation and polarization used on the left-hand side are computed on gross real annual earnings using an identification window of 20% of own inter-temporal earnings, an adjusted triweight weighting scheme, and including workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

Figure 5 plots the coefficient for the indicator for women in the regression also including indicators for tertiary graduates and workers in the South and Islands. It is remarkable that the degree of memory is crucial to assess the role of gender on inter-temporal polarization and its components. For older cohorts, while women were more concentrated into big-sized groups than men in terms of income at age 45, they were less concentrated than men in terms of income trajectories. However, if we look at the same graph excluding zero earners (Figure B.6), the stronger cross-sectional identification of women disappears: it is entirely due to the zeros at age 45 which were more frequent among women. This explains why we see in Figure 5 that the identification gap is closing for subsequent cohorts, consistently with increasing participation and stability of women in the labour market.

With or without zeros, the alienation component is on average larger for men than for women, with a gap rapidly widening across cohorts. A possible explanation is linked to the nature of the rising inequality we see in Figure 4: if the rise is due to a right tail getting

further away, and in that tail there are mostly men, then we should see an increase in alienation for men faster than that of women. If women are less identified and less alienated, they clearly also show lower levels of effective antagonism. The gap is reducing across cohorts but gender differences remain also for young cohorts. No statistically significant differences in effective antagonism emerge comparing different levels of memory, unless we include zero earners: because of the role of non-employment for the identification process, the resulting effective antagonism is stronger for women than for men when it is computed at age 45 with no memory. Using income trajectories, instead, reduces the gender gap in antagonism.

Figure 6: Education differences in inter-temporal polarization

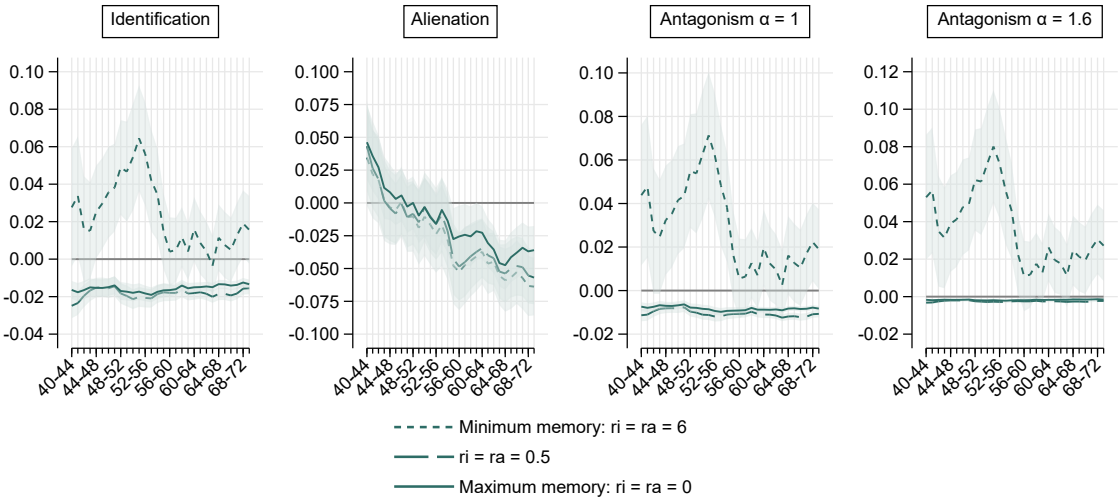


Note: The figure plots the coefficient of the indicator variable for tertiary education in a linear regression of inter-temporal identification, alienation, and effective antagonism on three indicator variables for women, tertiary educated, and workers in the South or Islands of Italy. The regressions are performed separately for each cohort of birth and level of the parameter of memory r — 0, 0.5, 6 —, and $r_I = r_A$ to make the results manageable. Consider that higher r means weaker memory. The confidence intervals plotted are at 95% confidence level computed using robust standard errors. The values of individual inter-temporal identification, alienation and polarization used on the left-hand side are computed on gross real annual earnings using an identification window of 20% of own inter-temporal earnings, an adjusted triweight weighting scheme, and including workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

With regard to education differences, Figure 6 shows that tertiary graduates are less identified but far more alienated than non-graduates, consistently with a concentration of more educated workers in the upper tail of the distribution. The differences in identification are present both in terms of income at age 45 and of income trajectories, and the identification gap seems to be larger if measured only in the last year ($r_I = 6$), as it should be the case if tertiary education gives an advantage in terms of faster career progression. The large variance of the identification gap for oldest cohorts comes from the presence of graduated women with zero earnings for childcare reasons. If we look at Figure B.7 where zero earnings are excluded, the coefficients for the old cohorts are in line with the overall pattern of an existing but closing over time gap between graduates and non-graduates. The resulting effective antagonism appears to be slightly larger for non-graduates, driven by their stronger group identity counterbalancing the smaller alienation. However, the gap

for the last cohorts is almost zero, suggesting that tertiary graduates are less polarized in the distribution than before.

Figure 7: Geographic differences in inter-temporal polarization



Note: The figure plots the coefficient of the indicator variable for working in the South or Islands of Italy in a linear regression of inter-temporal identification, alienation, and effective antagonism on three indicator variables for women, tertiary educated, and workers in the South or Islands of Italy. The regressions are performed separately for each cohort of birth and level of the parameter of memory r — 0, 0.5, 6 —, and $r_I = r_A$ to make the results manageable. Consider that higher r means weaker memory. The confidence intervals plotted are at 95% confidence level computed using robust standard errors. The values of individual inter-temporal identification, alienation and polarization used on the left-hand side are computed on gross real annual earnings using an identification window of 20% of own inter-temporal earnings, an adjusted triweight weighting scheme, and including workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

Also for geographic differences, there is a large role for the zeros when there is no memory: in Figure 7, workers in the South and Islands seem to be part of bigger sized income groups at age 45 with respect to the other geographic areas, at least for some cohorts. However, in Figure B.8, in which zero earners are excluded, we don't see anymore this identification advantage for the South, while we see a lower or equal level of group identity depending on the cohort. The alienation component, while very similar in the past for all the areas, seems to be smaller in the South for younger cohorts. The effective antagonism is therefore weaker on average for workers in the South and Islands due to lower identification and alienation. Looking at the role of memory, when excluding the zeros geographical differences appear not to be linked to the level of memory: this suggests a relevant degree of persistence in earnings differences linked to the area of work. As we have seen for gender differences, also for the geographical area the non-employed play a crucial role in making workers in the South and Islands more polarized at age 45 than the other groups.

6 Discussion and Conclusions

We start from a well-established literature on income polarization modelled as the interaction between group identity and income distance (Esteban and Ray, 1994; Duclos et al.,

2004) to develop an index of inter-temporal polarization to explicitly incorporate the time dimension in polarization analysis. Our main claim is that cross-sectional indices fail in properly measuring identification and alienation in presence of income mobility because they do not account for the duration of individuals' proximity or distance that matters for building ties. We introduce the concept of inter-temporal distance based on discounting past income differences through a parameter of memory. By simply replacing the cross-sectional income distance with this more complex notion of reciprocal proximity or remoteness, our index measures the concentration around poles of income trajectories rather than point-in-time incomes, allowing income dynamics to mediate the identification-alienation mechanism.

By applying the index to a sample of about 22,000 workers in Italy covering the years 1975–2018, we provide a concrete example of how much the measure of income polarization is sensitive to the longitudinal perspective. We adopt a cohort approach and follow workers every year between ages 35 and 45, computing income polarization and its two components—identification and alienation—within each cohort separately. First of all, we document for cohorts of Italian workers a long-term trend of decreasing identification and increasing alienation, regardless of the degree of memory. The decline in identification prevails over the rise in alienation, leading to a picture of falling polarization. The role of memory is relevant for the level of identification: incorporating past income differences leads to a lower rate of inclusion of neighbours into one's identification window, uncovering that it is more rare to be close in trajectory than in mid-career earnings. On the other hand, a stronger memory mitigates alienation by reducing large income distances between people, suggesting that the high cross-sectional inequality in mid-career is due to income trajectories that diverge over time.

We also uncover in our application that the effect of memory is not constant across cohorts: for some of them, polarization in mid-career is stronger than that in income trajectory, but for others the two are not so distant: this can be a useful indicator on how persistent income proximity and distance are along people's careers.

Heterogeneity analysis, exploiting the linearity of the index, sheds some light on which part of the population can be defined as more antagonistic. Women seem to be less concentrated in big-sized groups than men, and also less alienated from the rest of the distribution, showing a lower level of overall antagonism with respect to men. This difference is stronger in middle-career than in income trajectory and is reversed when zero incomes are included for old cohorts: the presence of a big group of women in non-employment makes them more polarized than men. A similar situation emerges for workers in the South and Islands, for whom the level of antagonism is lower than the rest of the population as long as the zero earners are excluded.

These results lead us to discuss the possible consequences of different levels of antagonism across social groups: the literature on income polarization started by Esteban and Ray (1994) was born to measure conflict, defined as a situation in which different social

groups with opposing interests suffer losses to increase the probability of obtaining their preferred outcome (Esteban and Ray, 1999). Conflict is therefore a source of resource dissipation, and we can think of effective antagonism as a correlate of bargaining power in a conflict society. If there are groups that are less polarised than others, in the sense that they are less clustered in relatively large and isolated groups, they may represent the part of the population that is deprived of a political voice and the opportunity to claim better conditions.

References

Bossert, W., S. R. Chakravarty, and C. D'Ambrosio (2012). Poverty and time. *Journal of Economic Inequality 10*(2), 145–162.

Calvo, C. and S. Dercon (2009). Chronic poverty and all that. *Poverty dynamics: interdisciplinary perspectives*, 29–58.

Duclos, J.-Y., J. Esteban, and D. Ray (2004). Polarization: concepts, measurement, estimation. *Econometrica 72*(6), 1737–1772.

Esteban, J., C. Gradín, and D. Ray (2007). An extension of a measure of polarization, with an application to the income distribution of five OECD countries. *Journal of Economic Inequality 5*(1), 1–19.

Esteban, J. and D. Ray (1994). On the measurement of polarization. *Econometric*, 819–851.

Esteban, J. and D. Ray (1999). Conflict and distribution. *Journal of Economic Theory 87*(2), 379–415.

Foster, J. E. and M. C. Wolfson (2010). Polarization and the decline of the middle class: Canada and the US. *Journal of Economic Inequality 8*(2), 247–273.

Hoy, M., B. S. Thompson, and B. Zheng (2012). Empirical issues in lifetime poverty measurement. *The Journal of Economic Inequality 10*(2), 163–189.

Hoy, M. and B. Zheng (2011). Measuring lifetime poverty. *Journal of Economic Theory 146*(6), 2544–2562.

Manacorda, M. (2004). Can the Scala Mobile explain the fall and rise of earnings inequality in Italy? A semiparametric analysis, 1977–1993. *Journal of Labor Economics 22*(3), 585–613.

Wolfson, M. C. (1994). When inequalities diverge. *American Economic Review 84*(2), 353–358.

Wolfson, M. C. (1997). Divergent inequalities: theory and empirical results. *Review of Income and Wealth 43*(4), 401–421.

Appendix A Proof of Proposition 1

Proof of Proposition 1 exploits the following lemmas.[8]

Lemma 2. *For any twice differentiable function $f(\boldsymbol{d})$, the conditions $\frac{\partial^2 f(\boldsymbol{d})}{\partial d_s \partial d_t} = 0 \; \forall s \neq t$, $s, t \in 1, 2, ..., T$ are satisfied if and only if $f(\boldsymbol{d})$ can be written as a sum $f(\boldsymbol{d}) = f_1(d_1) + f_2(d_2) + ... + f_T(d_T)$ for some f_t that are twice differentiable.*

Lemma 3. *For any twice differentiable function $f_t(d_t)$, the conditions $f_t''(d_t) = 0$ are satisfied $\forall\, t \in 1, 2, ..., T$ if and only if $f_t(d_t)$ can be written as a linear function $f_t(d_t) = \phi_t^1 d_t + \phi_t^0$, $\forall\, t$.*

Proposition 1 can therefore be obtained as follows.

Proof. Given Monotonicity, $D(\mathbf{y}, \mathbf{y'})$ is increasing in each period income distance $|y_t - y_t'|$. Therefore, there exists a continuous and increasing function f such that

$$D(\mathbf{y}, \mathbf{y'}) = f\left(|y_1 - y_1'|, |y_2 - y_2'|, ..., |y_T - y_T'|\right) \tag{1}$$

Given Independence, Lemma 2 implies that $D(\mathbf{y}, \mathbf{y'})$ is additively separable in period distance $|y_t - y_t'|$ for some continuous and positive functions f_1, f_2, ..., f_T:

$$D(\mathbf{y}, \mathbf{y'}) = f_1(|y_1 - y_1'|) + f_2(|y_2 - y_2'|) + ... + f_T(|y_T - y_T'|). \tag{2}$$

Independence also requires $\frac{\partial^2 D(\mathbf{y}, \mathbf{y'})}{\partial |y_t - y_t'|^2} = 0$, so

$$\frac{\partial^2 D(\mathbf{y}, \mathbf{y'})}{\partial |y_t - y_t'|^2} = f_t''(|y_t - y_t'|) = 0. \tag{3}$$

Lemma 3 imposes the linear functional form to each $f_t(|y_t - y_t'|)$, leading to:

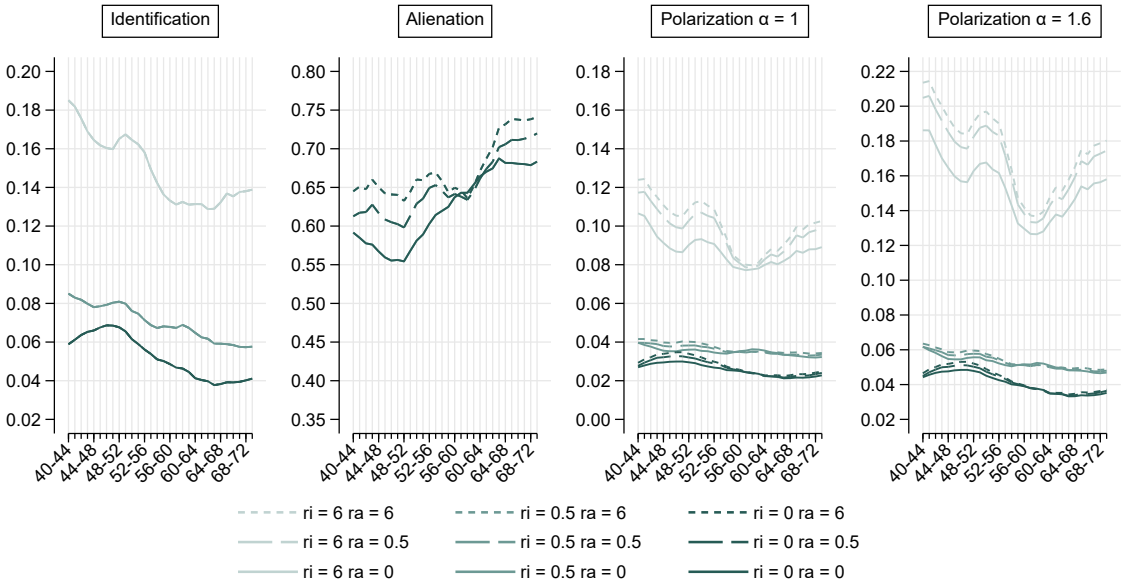
$$D(\mathbf{y}, \mathbf{y'}) = \phi_1^1 |y_1 - y_1'| + \phi_t^0 + \phi_2^1 |y_2 - y_2'| + \phi_2^0 + ... + \phi_T^1 |y_T - y_T'| + \phi_T^0. \tag{4}$$

$\forall\, t$, Monotonicy ensures $\phi_t^1 \geq 0$, while Relevance requires $\phi_t^0 = 0$. Normalization finally imposes $\sum_{t=1}^{T} \phi_t^1 = 1$. □

[8] Lemma 1 is derived in Hoy and Zheng (2011, pp.2560–2561)

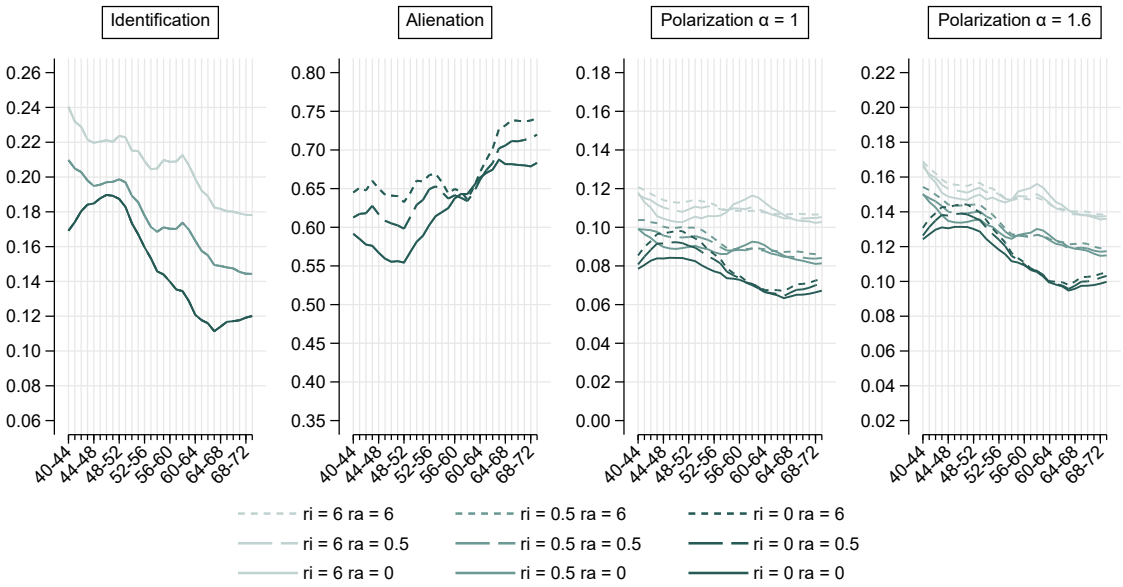
Appendix B Additional figures and tables

Figure B.1: Inter-temporal polarization by cohort – triangular weighting



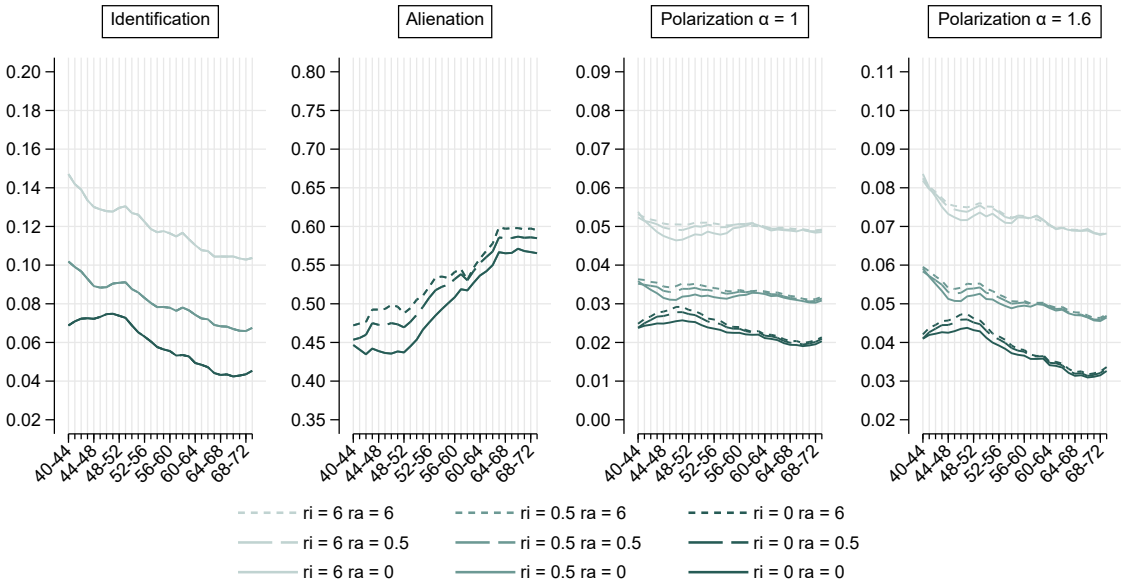
Note: The figure plots the value of average inter-temporal identification, alienation and polarization of gross real annual earnings for all the possible combinations of three values $(0, 0.5, 6)$ of the discount rates of past differences for identification r_I and for alienation r_A. The identification window width is defined as 20% of own inter-temporal earnings, and a triangular weighting scheme is applied to weight observations inside one's identification window. The sample includes employees in the private sector, and the indices are computed separately for thirty five-year rolling cohorts of birth, from 1940-1944 to 1969-1973 among workers observed every year from age 35 to 45. Workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

Figure B.2: Inter-temporal polarization by cohort – rectangular weighting



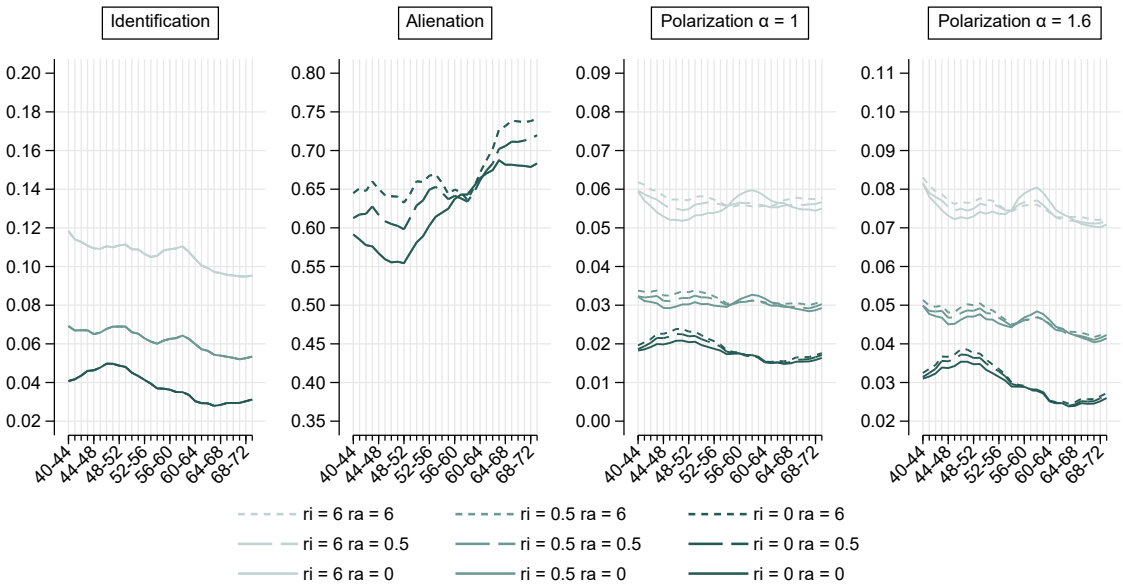
Note: The figure plots the value of average inter-temporal identification, alienation and polarization of gross real annual earnings for all the possible combinations of three values $(0, 0.5, 6)$ of the discount rates of past differences for identification r_I and for alienation r_A. The identification window width is defined as 20% of own inter-temporal earnings, and a rectangular weighting scheme is applied to weight observations inside one's identification window. The sample includes employees in the private sector, and the indices are computed separately for thirty five-year rolling cohorts of birth, from 1940-1944 to 1969-1973 among workers observed every year from age 35 to 45. Workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

Figure B.3: Inter-temporal polarization by cohort – only positive earnings



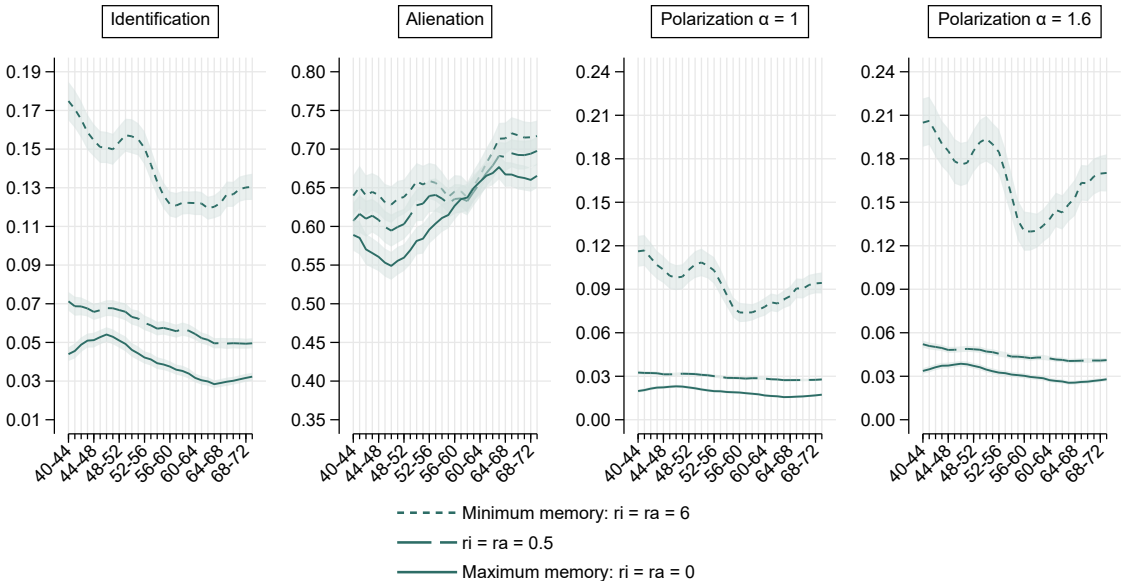
Note: The figure plots the value of average inter-temporal identification, alienation and polarization of gross real annual earnings for all the possible combinations of three values $(0, 0.5, 6)$ of the discount rates of past differences for identification r_I and for alienation r_A. The identification window width is defined as 20% of own inter-temporal earnings, and an adjusted triweight weighting scheme is applied to weight observations inside one's identification window. The sample includes employees in the private sector, and the indices are computed separately for thirty five-year rolling cohorts of birth, from 1940-1944 to 1969-1973 among workers observed every year from age 35 to 45. Workers with even one year of zero are excluded. *Source*: AD-SILC data 1975–2018.

Figure B.4: Inter-temporal polarization by cohort – absolute threshold



Note: The figure plots the value of average inter-temporal identification, alienation and polarization of gross real annual earnings for all the possible combinations of three values $(0, 0.5, 6)$ of the discount rates of past differences for identification r_I and for alienation r_A. The identification window width is defined as 20% of the mean inter-temporal earnings and an adjusted triweight weighting scheme is applied to weight observations inside one's identification window. The sample includes employees in the private sector, and the indices are computed separately for thirty five-year rolling cohorts of birth, from 1940-1944 to 1969-1973 among workers observed every year from age 35 to 45. Workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

Figure B.5: Inter-temporal polarization by cohort with bootstrap CI



Note: The figure plots the value of average inter-temporal identification, alienation and polarization of gross real annual earnings for the three values of memory $r = r_I = r_A = 0, 0.5, 6$ for both identification and alienation. Consider that higher r means weaker memory. The identification window width is defined as 20% of own inter-temporal earnings, and a triangular weighting scheme is applied to weight observations inside one's identification window. Normal-based confidence intervals at 95% confidence level are computed using 1,000 bayesian bootstrap repetitions and reported through the shaded areas. The sample includes employees in the private sector and the indices are computed separately for thirty five-year rolling cohorts of birth, from 1940-1944 to 1969-1973 among workers observed every year from age 35 to 45. Workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

Table B.1: Summary statistics

		Annual earnings (€)								
Cohort	N	Mean	SD	p10	p50	p90	Zeros	Women	Tertiary	South
1940-1944	2,952	22,137	12,259	3,614	21,997	36,210	6.0	29.9	2.7	20.1
1941-1945	2,837	23,355	13,190	6,006	22,960	36,574	4.5	30.4	3.8	21.4
1942-1946	2,983	22,941	13,760	3,091	22,427	37,994	6.0	31.7	2.8	20.6
1943-1947	3,126	23,716	13,015	6,173	23,338	38,813	4.2	30.8	3.7	21.9
1944-1948	3,235	24,975	13,577	7,770	23,600	42,616	2.7	31.4	3.0	23.3
1945-1949	3,322	25,561	13,612	8,699	24,907	39,277	4.1	32.6	4.6	20.7
1946-1950	3,414	25,381	13,314	8,768	24,609	40,502	3.7	33.9	3.2	22.7
1947-1951	3,335	25,678	13,846	7,449	25,027	41,121	4.7	31.2	3.9	19.8
1948-1952	3,238	25,728	13,696	8,079	24,751	41,960	4.4	30.8	3.5	27.3
1949-1953	3,186	25,253	14,334	6,352	24,317	43,740	4.2	33.6	4.5	26.0
1950-1954	3,178	25,629	14,475	5,736	24,824	41,928	6.1	36.0	4.6	18.8
1951-1955	3,240	25,929	14,995	7,181	25,189	42,359	4.3	31.2	4.5	22.8
1952-1956	3,320	26,023	14,907	7,419	25,009	45,262	4.7	35.4	2.7	18.2
1953-1957	3,402	25,585	14,281	6,326	24,191	43,574	4.2	39.0	4.4	20.8
1954-1958	3,490	26,294	14,749	7,684	25,398	43,088	4.8	36.2	4.2	23.6
1955-1959	3,555	26,557	15,424	9,879	24,536	43,765	3.3	35.0	4.4	18.7
1956-1960	3,573	26,388	16,360	5,281	25,238	45,993	4.9	35.1	4.7	21.6
1957-1961	3,648	25,295	14,183	7,274	24,157	44,686	4.2	35.9	6.7	28.6
1958-1962	3,828	26,282	15,694	8,902	23,652	47,507	3.8	40.8	6.1	20.8
1959-1963	4,028	24,160	15,148	5,805	22,907	43,873	4.3	36.2	5.2	28.0
1960-1964	4,282	25,002	15,858	6,403	23,632	44,232	5.3	35.8	6.8	25.0
1961-1965	4,584	26,176	15,587	7,752	24,737	46,530	3.7	36.2	6.4	25.5
1962-1966	4,854	24,040	15,546	5,803	22,893	42,133	5.2	41.4	7.5	22.5
1963-1967	5,048	24,749	16,611	5,165	22,800	44,776	4.8	36.0	6.6	24.2
1964-1968	5,171	25,185	16,796	6,276	23,206	45,721	4.3	42.5	6.7	26.4
1965-1969	5,258	27,501	18,941	7,638	24,571	48,408	4.3	39.1	8.5	23.7
1966-1970	5,308	25,352	17,042	6,165	23,716	44,052	4.1	40.6	7.4	25.7
1967-1971	5,285	24,897	15,895	6,622	23,670	42,786	5.2	42.5	11.6	28.2
1968-1972	5,243	26,060	17,518	6,878	23,713	47,433	4.4	43.8	14.1	27.3
1969-1973	5,173	26,036	17,001	5,671	24,284	46,201	4.9	45.2	14.8	24.4
All	26,645	25,262	15,500	6,584	23,894	43,332	4.6	37.2	6.9	23.7

Note: The table reports the number of workers and summary statistics for thirty five-year rolling cohorts of birth (1940-1944 to 1969-1973) of employees in the private sector in Italy. The workers are observed every year from age 35 to 45. Workers with zero earnings for at most five years are included. Annual Earnings are real (2015 price level) and gross of personal income taxes and social contributions. The percentage of zero earnings, women, tertiary graduates, workers in the South or Islands of Italy are reported. We use EU-SILC sample weights. *Source*: AD-SILC data 1975–2018.

Table B.2: Summary statistics - only positive earnings

		Annual earnings (€)							
Cohort	N	Mean	SD	p10	p50	p90	Women	Tertiary	South
1940-1944	2,362	25,520	11,196	14,572	23,898	38,412	24.1	3.2	18.2
1941-1945	2,278	25,567	10,997	14,364	24,113	38,300	26.0	3.4	18.8
1942-1946	2,439	26,381	11,335	15,145	24,813	39,198	26.6	3.5	19.7
1943-1947	2,590	27,202	12,127	15,595	25,321	40,571	26.0	4.1	20.8
1944-1948	2,706	27,375	12,030	15,603	25,611	41,121	26.7	3.8	21.5
1945-1949	2,800	27,700	12,118	15,766	25,976	41,458	27.1	3.8	21.7
1946-1950	2,872	28,101	12,332	16,318	26,206	42,176	27.2	3.8	21.5
1947-1951	2,793	28,333	12,399	16,259	26,466	42,707	27.4	4.3	21.4
1948-1952	2,698	28,289	12,153	16,218	26,521	42,679	28.1	4.0	21
1949-1953	2,636	28,545	12,562	16,347	26,443	43,323	29.1	4.4	19.8
1950-1954	2,616	28,977	12,908	16,602	26,738	44,495	29.4	5.0	18.8
1951-1955	2,684	29,100	13,405	16,258	26,781	44,891	31.1	5.2	18.1
1952-1956	2,733	29,251	13,729	16,156	26,797	45,593	31.3	4.9	17.2
1953-1957	2,811	29,377	13,990	15,802	26,831	46,141	32.2	5.2	17.0
1954-1958	2,874	29,423	14,171	15,226	26,932	46,413	32.7	5.1	17.1
1955-1959	2,934	29,230	14,320	14,702	26,570	46,524	33.7	5.3	17.5
1956-1960	2,934	29,423	14,604	14,541	26,519	47,530	33.2	5.9	18.5
1957-1961	2,990	29,217	14,731	13,942	26,243	47,707	34.4	6.2	19.3
1958-1962	3,115	29,310	14,750	14,062	26,252	47,845	34.1	6.3	19.2
1959-1963	3,246	28,924	14,850	13,396	25,879	47,689	34.3	6.4	19.9
1960-1964	3,429	28,801	14,968	12,916	25,857	47,790	34.7	6.4	20.6
1961-1965	3,657	28,448	14,914	12,235	25,624	47,259	34.6	6.7	21.1
1962-1966	3,896	28,317	15,200	11,875	25,507	47,175	35.3	7.1	21.1
1963-1967	4,068	28,370	15,847	11,478	25,486	47,597	36.5	8.0	21.5
1964-1968	4,225	28,725	16,018	11,706	25,779	47,963	37.0	8.9	21.8
1965-1969	4,324	28,762	16,105	11,783	25,811	47,697	37.9	9.7	22.2
1966-1970	4,405	28,861	16,367	11,782	25,935	47,886	39.5	10.3	21.7
1967-1971	4,380	28,940	16,256	11,958	26,056	48,022	40.2	11.7	22.7
1968-1972	4,356	28,944	16,140	12,086	25,977	48,328	41.0	13.2	22.9
1969-1973	4,279	29,093	16,159	12,273	26,054	48,596	41.7	15.0	22.2
All	21,849	28,520	14,449	13,547	25,972	45,675	33.5	7.0	20.5

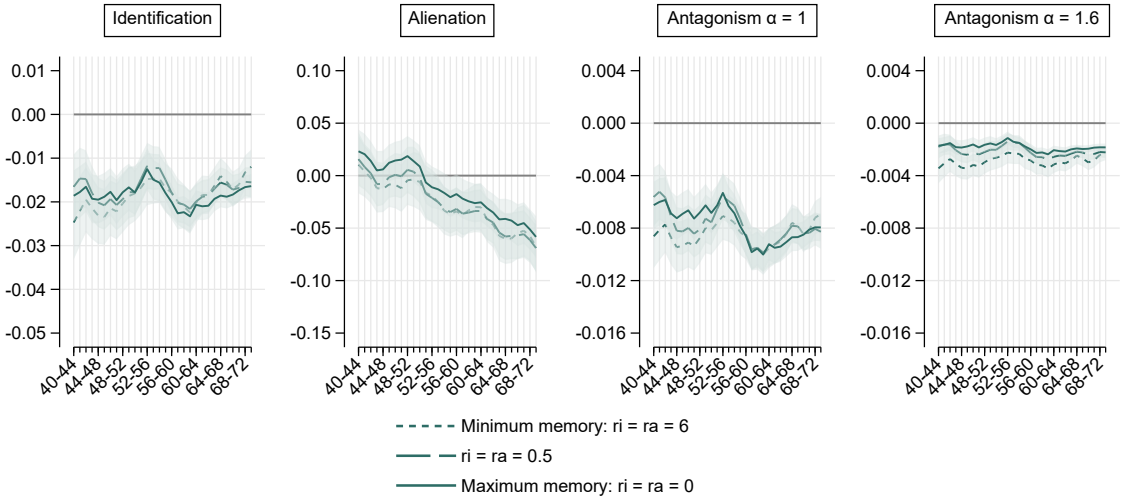
Note: The table reports the number of workers and summary statistics for thirty five-year rolling cohorts of birth (1940-1944 to 1969-1973) of employees in the private sector in Italy. The workers are observed every year from age 35 to 45. Only workers with positive earnings every year are included. Annual Earnings are real (2015 price level) and gross of personal income taxes and social contributions. The percentage of zero earnings, women, tertiary graduates, workers in the South or Islands of Italy are reported. We use EU-SILC sample weights. *Source*: AD-SILC data 1975–2018.

Table B.3: Inter-temporal polarization index with bootstrap standard errors

	Identification			Alienation			Polarization $\alpha = 1$			Polarization $\alpha = 1.6$		
Cohort	$r = 0$	$r = 0.5$	$r = 6$	$r = 0$	$r = 0.5$	$r = 6$	$r = 0$	$r = 0.5$	$r = 6$	$r = 0$	$r = 0.5$	$r = 6$
1940-1944	0.044	0.071	0.175	0.589	0.607	0.640	0.020	0.033	0.116	0.034	0.052	0.205
	(0.002)	(0.002)	(0.005)	(0.010)	(0.012)	(0.013)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1941-1945	0.046	0.069	0.171	0.585	0.616	0.651	0.020	0.032	0.117	0.035	0.051	0.206
	(0.002)	(0.003)	(0.005)	(0.011)	(0.013)	(0.014)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1942-1946	0.049	0.069	0.165	0.570	0.610	0.640	0.021	0.032	0.112	0.036	0.050	0.198
	(0.002)	(0.003)	(0.005)	(0.010)	(0.012)	(0.013)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.009)
1943-1947	0.051	0.068	0.159	0.566	0.614	0.645	0.022	0.032	0.107	0.037	0.049	0.190
	(0.002)	(0.002)	(0.005)	(0.010)	(0.011)	(0.012)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1944-1948	0.051	0.066	0.155	0.561	0.608	0.641	0.022	0.031	0.104	0.037	0.048	0.185
	(0.002)	(0.002)	(0.004)	(0.009)	(0.011)	(0.012)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1945-1949	0.053	0.067	0.151	0.553	0.599	0.631	0.023	0.031	0.099	0.038	0.048	0.178
	(0.002)	(0.002)	(0.004)	(0.009)	(0.011)	(0.012)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1946-1950	0.054	0.068	0.151	0.549	0.595	0.628	0.023	0.031	0.098	0.039	0.049	0.176
	(0.002)	(0.002)	(0.004)	(0.009)	(0.011)	(0.012)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)	(0.008)
1947-1951	0.053	0.068	0.150	0.556	0.599	0.635	0.023	0.032	0.099	0.038	0.049	0.177
	(0.002)	(0.002)	(0.004)	(0.009)	(0.010)	(0.012)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)	(0.007)
1948-1952	0.051	0.067	0.154	0.560	0.603	0.638	0.022	0.032	0.103	0.037	0.049	0.185
	(0.002)	(0.002)	(0.004)	(0.009)	(0.010)	(0.011)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)	(0.007)
1949-1953	0.049	0.066	0.157	0.569	0.615	0.647	0.022	0.031	0.107	0.036	0.048	0.191
	(0.002)	(0.002)	(0.004)	(0.010)	(0.011)	(0.013)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1950-1954	0.046	0.063	0.157	0.581	0.627	0.658	0.021	0.031	0.109	0.035	0.047	0.194
	(0.002)	(0.002)	(0.005)	(0.009)	(0.011)	(0.012)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1951-1955	0.044	0.062	0.155	0.584	0.630	0.654	0.020	0.031	0.106	0.033	0.047	0.190
	(0.002)	(0.002)	(0.004)	(0.010)	(0.011)	(0.012)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)	(0.008)
1952-1956	0.042	0.060	0.151	0.596	0.639	0.659	0.020	0.030	0.103	0.033	0.046	0.185
	(0.002)	(0.002)	(0.004)	(0.009)	(0.011)	(0.012)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1953-1957	0.041	0.059	0.142	0.604	0.641	0.656	0.020	0.030	0.095	0.032	0.045	0.171
	(0.002)	(0.002)	(0.004)	(0.009)	(0.011)	(0.012)	(0.001)	(0.001)	(0.005)	(0.001)	(0.001)	(0.008)
1954-1958	0.039	0.057	0.133	0.611	0.636	0.648	0.019	0.029	0.086	0.031	0.044	0.155
	(0.002)	(0.002)	(0.004)	(0.009)	(0.010)	(0.011)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)	(0.008)
1955-1959	0.039	0.058	0.126	0.615	0.629	0.638	0.019	0.029	0.078	0.031	0.044	0.138
	(0.001)	(0.002)	(0.003)	(0.009)	(0.010)	(0.011)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)	(0.007)
1956-1960	0.038	0.057	0.122	0.627	0.636	0.645	0.019	0.029	0.074	0.03	0.043	0.130
	(0.001)	(0.002)	(0.003)	(0.009)	(0.010)	(0.011)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.007)
1957-1961	0.036	0.056	0.121	0.635	0.636	0.645	0.018	0.028	0.074	0.03	0.043	0.130
	(0.001)	(0.002)	(0.003)	(0.009)	(0.009)	(0.010)	(0.000)	(0.001)	(0.003)	(0.001)	(0.001)	(0.007)
1958-1962	0.035	0.057	0.122	0.637	0.633	0.637	0.018	0.029	0.074	0.029	0.043	0.130
	(0.001)	(0.002)	(0.003)	(0.009)	(0.009)	(0.010)	(0.000)	(0.001)	(0.003)	(0.001)	(0.001)	(0.006)
1959-1963	0.034	0.056	0.122	0.650	0.645	0.652	0.018	0.029	0.076	0.029	0.043	0.134
	(0.001)	(0.002)	(0.003)	(0.009)	(0.009)	(0.010)	(0.000)	(0.001)	(0.003)	(0.001)	(0.001)	(0.006)
1960-1964	0.032	0.054	0.122	0.658	0.656	0.665	0.017	0.028	0.078	0.027	0.042	0.138
	(0.001)	(0.002)	(0.003)	(0.008)	(0.009)	(0.010)	(0.000)	(0.000)	(0.003)	(0.001)	(0.001)	(0.006)
1961-1965	0.030	0.052	0.122	0.666	0.669	0.682	0.016	0.028	0.081	0.027	0.042	0.145
	(0.001)	(0.001)	(0.003)	(0.008)	(0.009)	(0.010)	(0.000)	(0.000)	(0.003)	(0.001)	(0.001)	(0.006)
1962-1966	0.030	0.051	0.119	0.669	0.679	0.696	0.016	0.028	0.080	0.026	0.041	0.143
	(0.001)	(0.001)	(0.003)	(0.008)	(0.009)	(0.010)	(0.000)	(0.000)	(0.003)	(0.001)	(0.001)	(0.006)
1963-1967	0.028	0.050	0.120	0.677	0.691	0.713	0.016	0.027	0.083	0.026	0.041	0.149
	(0.001)	(0.001)	(0.003)	(0.008)	(0.009)	(0.010)	(0.000)	(0.000)	(0.003)	(0.001)	(0.001)	(0.006)
1964-1968	0.029	0.050	0.122	0.667	0.690	0.714	0.016	0.027	0.085	0.026	0.041	0.153
	(0.001)	(0.001)	(0.003)	(0.008)	(0.009)	(0.010)	(0.000)	(0.000)	(0.003)	(0.001)	(0.001)	(0.006)
1965-1969	0.030	0.049	0.127	0.667	0.695	0.720	0.016	0.027	0.091	0.026	0.041	0.163
	(0.001)	(0.001)	(0.003)	(0.008)	(0.009)	(0.011)	(0.000)	(0.000)	(0.003)	(0.001)	(0.001)	(0.006)
1966-1970	0.030	0.050	0.127	0.664	0.692	0.718	0.016	0.027	0.090	0.026	0.041	0.163
	(0.001)	(0.001)	(0.003)	(0.008)	(0.010)	(0.011)	(0.000)	(0.000)	(0.003)	(0.001)	(0.001)	(0.006)
1967-1971	0.031	0.049	0.129	0.663	0.692	0.715	0.016	0.027	0.093	0.027	0.041	0.168
	(0.001)	(0.001)	(0.003)	(0.008)	(0.010)	(0.011)	(0.000)	(0.000)	(0.004)	(0.001)	(0.001)	(0.006)
1968-1972	0.032	0.049	0.130	0.660	0.694	0.715	0.017	0.028	0.094	0.027	0.041	0.170
	(0.001)	(0.001)	(0.003)	(0.008)	(0.009)	(0.010)	(0.000)	(0.000)	(0.004)	(0.001)	(0.001)	(0.006)
1969-1973	0.032	0.050	0.130	0.665	0.698	0.717	0.017	0.028	0.094	0.028	0.041	0.170
	(0.001)	(0.001)	(0.003)	(0.008)	(0.009)	(0.010)	(0.000)	(0.000)	(0.004)	(0.001)	(0.001)	(0.007)

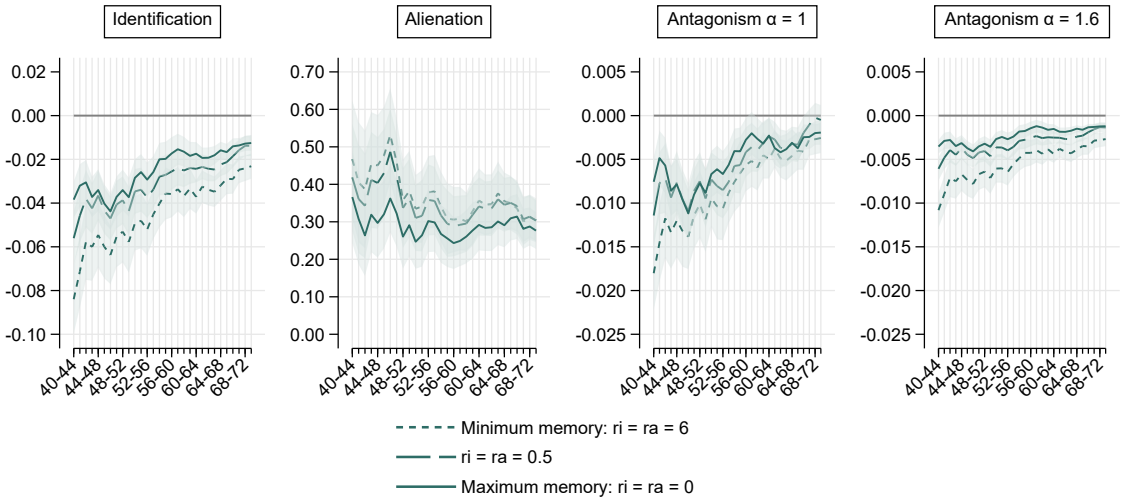
Note: The table reports the values and standard errors of average inter-temporal identification, alienation and polarization of gross real annual earnings for the three values of memory $r = r_I = r_A = 0, 0.5, 6$ for both identification and alienation. Consider that higher r means weaker memory. The identification window width is defined as 20% of own inter-temporal earnings, and a triweight weighting scheme is applied to weight observations inside one's identification window. The standard errors are computed using 1,000 bayesian bootstrap repetitions. The sample includes employees in the private sector and the indices are computed separately for thirty five-year rolling cohorts of birth, from 1940-1944 to 1969-1973 among workers observed every year from age 35 to 45. Workers with zero earnings for at most five years are included. *Source*: AD-SILC data 1975–2018.

Figure B.6: Gender differences in inter-temporal polarization — only positive earnings



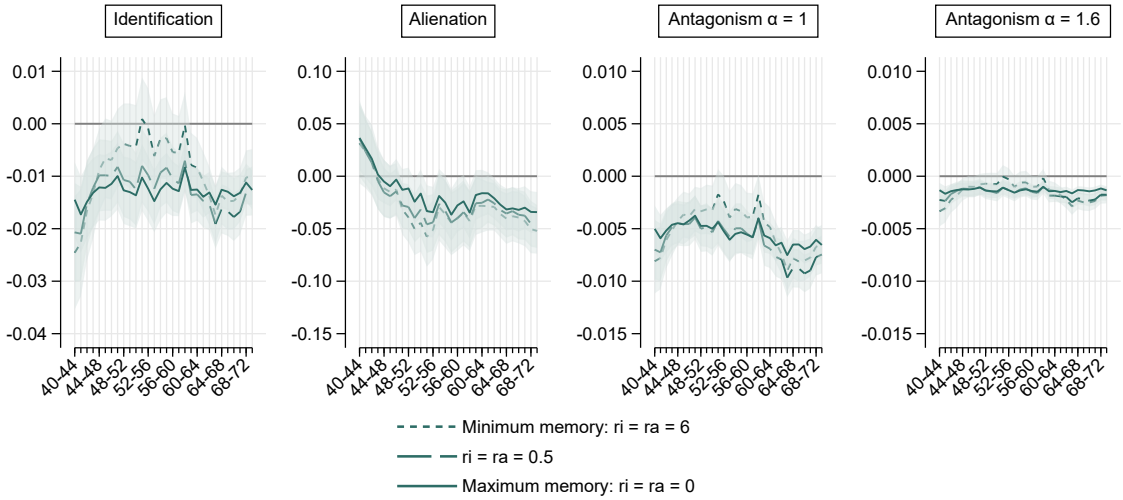
Note: The figure plots the coefficient of the indicator variable for women in a linear regression of inter-temporal identification, alienation, and effective antagonism on three indicator variables for women, tertiary educated, and workers in the South or Islands of Italy. The regressions are performed separately for each cohort of birth and level of parameter of memory r — 0, 0.5, 6 —, and $r_I = r_A$ to make the results manageable. Consider that higher r means weaker memory. The confidence intervals plotted are at 95% confidence level computed using robust standard errors. The values of individual inter-temporal identification, alienation and polarization used on the left-hand side are computed on gross real annual earnings using an identification window of 20% of own inter-temporal earnings, an adjusted triweight weighting scheme, and excluding workers with zero earnings in any year. *Source*: AD-SILC data 1975–2018.

Figure B.7: Education differences in polarization — only positive earnings



Note: The figure plots the coefficient of the indicator variable for tertiary education in a linear regression of inter-temporal identification, alienation, and effective antagonism on three indicator variables for women, tertiary educated, and workers in the South or Islands of Italy. The regressions are performed separately for each cohort of birth and level of parameter of memory r — 0, 0.5, 6 —, and $r_I = r_A$ to make the results manageable. Consider that higher r means weaker memory. The confidence intervals plotted are at 95% confidence level computed using robust standard errors. The values of individual inter-temporal identification, alienation and polarization used on the left-hand side are computed on gross real annual earnings using an identification window of 20% of own inter-temporal earnings, an adjusted triweight weighting scheme, and excluding workers with zero earnings in any year. *Source*: AD-SILC data 1975–2018.

Figure B.8: Geographic differences in polarization — only positive earnings



Note: The figure plots the coefficient of the indicator variable for working in the South or Islands of Italy in a linear regression of inter-temporal identification, alienation, and effective antagonism on three indicator variables for women, tertiary educated, and workers in the South or Islands of Italy. The regressions are performed separately for each cohort of birth and level of parameter of memory r — 0, 0.5, 6 —, and $r_I = r_A$ to make the results manageable. Consider that higher r means weaker memory. The confidence intervals plotted are at 95% confidence level computed using robust standard errors. The values of individual inter-temporal identification, alienation and polarization used on the left-hand side are computed on gross real annual earnings using an identification window of 20% of own inter-temporal earnings, an adjusted triweight weighting scheme, and excluding workers with zero earnings in any year. *Source*: AD-SILC data 1975–2018.